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Fractal trajectories in a numerical model of the upper Indian Ocean

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Abstract. A wind-driven numerical model of the Indian Ocean is used to examine the horizontal statistics of hundreds of passive tracers spread evenly over the model domain. The distribution covers several dynamically distinct regions, revealing a variety of Lagrangian behaviors associated with different geographic locations. In particular, a cluster of trajectories with scaling dimension as large as 1.3 exists throughout the equatorial zone. Spectral analysis of trajectory displacements indicates mixed Rossby-gravity waves are involved in the production of some fractal trajectories.

1 Introduction

Of recent interest in oceanography have been developments in physics and mathematics known as dynamical systems (Guckenheimer and Holmes, 1983; Lichtenberg and Lieberman, 1983) and fractal geometry (Mandelbrot, 1983). Attempts at using these concepts to further the understanding of oceanic behavior have been constrained for two reasons. The first being the simplifications (sometimes severe) required for analytical studies of nonlinear ocean dynamics that make direct comparison to the real ocean difficult. The second, and perhaps more important, is the lack of extensive field measurements. Oceans are so complex in structure and behavior that without good observational motivation, sophisticated theoretical studies are of limited use. Though application of these ideas to the analysis of atmospheric observations has met with some success, e.g., Zeng et al. (1991), there have been few clear oceanic results.

For example, drifter trajectories in the Kuroshio and Gulf Stream extensions have been shown to have fractal properties, with correlation dimension $\nu \simeq 1.3$ (Osborne et al., 1986; Brown and Smith, 1991). Those investigations were in dynamically similar regions, both being extensions of mid-latitude western boundary currents. Additional studies by Sanderson and Booth (1991) showed similar results. The physical mechanism responsible for the fractal structure of the trajectories remains unclear, but may be related to geostrophic turbulence as discussed in Osborne et al. (1989) and Provenzale et al. (1991). Dynamically dissimilar regions would probably give rise to different trajectory statistics.

Studying fractal dimension alone does not generally yield the underlying physics. However, the two-point correlation dimension ν does indicate how well a trajectory covers the plane and yields a quantitative characteristic that analytical models of oceanic transport must explain. For example, the colored noise model of surface oceanic motions proposed by Osborne et al. (1986) and Osborne and Caponio (1990) reproduces the fractal and spectral properties in the aforementioned studies.

Here, trajectories from a numerical model of the Indian Ocean are used to study the spatial distribution of the scaling dimension (described below). Four trajectories are highlighted and their dimension and Fourier spectrum are examined. Equatorial regions of high dimension are shown to be associated with Yanai waves which leads to the conclusion that the noninteger dimensions are produced by a mechanism different from colored noise.

2 Trajectory Generation and Analysis

In place of extensive observational information, a multi-layer reduced gravity model driven by climatological winds is used to generate trajectories. The spacing between like-variables on the model C-grid is $1/6°$ with a model time step of 15 minutes. Twenty model years are used for spin-up, after which the model currents and mesoscale structure have been validated to be consistent with the horizontal circulation in the Indian Ocean (Jensen, 1990). After spin-up, passive tracers (drifters) are initialized over the basin with a meridional and zonal spacing of 3°. At each time step the drifters are trans-

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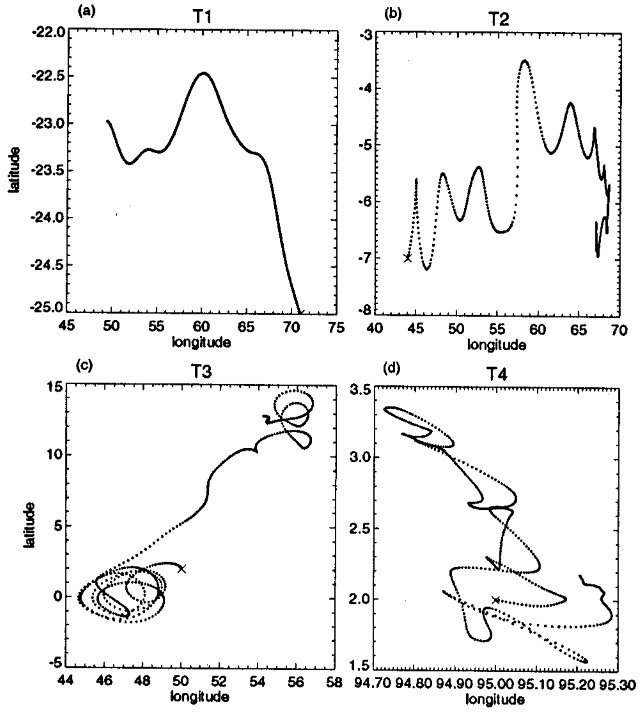

Fig. 1. Four typical trajectories. (a) T1 is smooth. $D \simeq 1.02$. (b) T2 has oscillations in the meridional direction but has a relatively steady zonal motion. $D \simeq 1.11$ (c) T3 appears to hop between vortices. $D \simeq 1.22$ (d) T4 is convoluted, possibly chaotic. $D \simeq 1.16$. Each point is separated by 12 model hours and the initial point is marked with an $\times$. The paths are for one model year.

lated according to a predictor-corrector scheme and their positions are recorded every 12 hours for one model year. Due to its restricted vertical structure the model cannot generate vertical mixing and only the horizontal component of tracer trajectories can be examined.

Some typical trajectories are shown in Fig. 1 and represent the four generic trajectory types identified in the model. Most trajectories have characteristics in common with more than one of these four, probably because they pass through regions with differing dynamics. The path statistics are now examined. In all the calculations of dimension the raw data is used without any filtering.

2.1 Dimension

The scaling dimension, H, of a scalar time series $X(t_i)$ is found by examining displacements at various time intervals:

$$\overline{|X(t+\lambda\Delta t) - X(t)|} = \lambda^H \overline{|\Delta X|}, \tag{1}$$

where there overline represents a time average. If such an exponent exists, then $X(t)$ is said to be self-affine. H is related to the Hausdorff fractal dimension as $D = 1/H$ (Mandelbrot, 1983). Measurements of H for the $x(t)$ and $y(t)$ associated with each trajectory are computed separately, as in Fig. 2. Since these trajectories are finite in

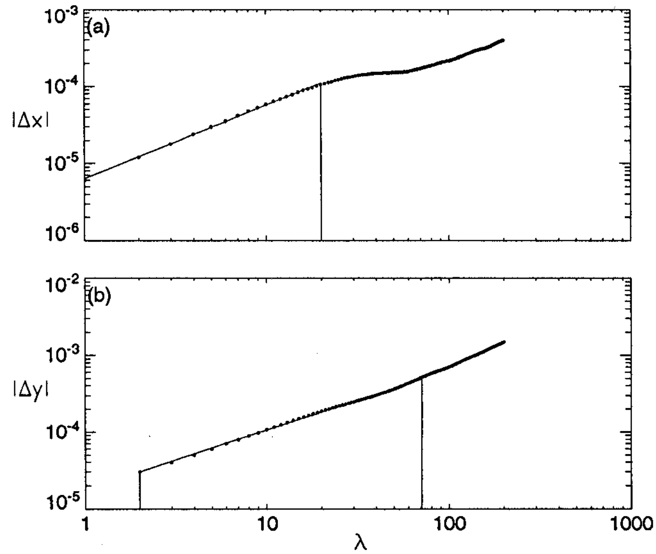

Fig. 2. The scaling function from Eq.(1) for trajectory T4 for (a) x displacements (b) y displacements. The line is a linear (95% confidence) fit and the vertical lines demark the limits of the scaling region. The limits can change depending on the accuracy of the fit demanded. The slope of the line is the scaling dimension, H.

length and temporal resolution, upper and lower cutoff scales for self-similarity exist. These cutoffs may also be physically significant as discussed in Provenzale et al. (1991). The average $(H_x + H_y)/2$ for each trajectory is used for the values in Fig. 3. Another method for characterizing the fractality of a trajectory is to find its correlation dimension (Grassberger and Procaccia, 1983), which yields very similar values.

The ν of a time series $X(t_i)$ is found from the correlation integral (Grassberger and Procaccia, 1983). Straightforward measurements of ν often result in estimates that are biased low, typically due to unevenly distributed data. This bias is overcome by "time-delay embedding" the data into a higher dimension by creating the N-vector $X^N(t_i) = (X(t_i), X(t_i+\tau), ..., X(t_i+(N-1)\tau)$, and finding the correlation integral in N-space

$$C^N(\epsilon) = (M^2 - M)^{-1} \sum_{i,j=1}^{M} H(\epsilon - \parallel X^N(t_i) - X^N(t_j) \parallel)$$

$$\sim \epsilon^{\nu_N}. \tag{2}$$

The dimension is then $\nu = \lim_{N\to\infty} \nu_N$. Here, the time-delay τ is chosen to be the typical time for the first zero-crossing of the trajectory autocorrelation function (10 days). A standard analysis is performed on all the trajectories (due to their large number) using $N = 20$. Most measurements have converged before $N = 13$. We have measured both D and ν for all the trajectories in this study and found virtually identical values for both dimensions.

Fig. 3 shows that trajectories along the equator have relatively high dimension and the dimensions generally

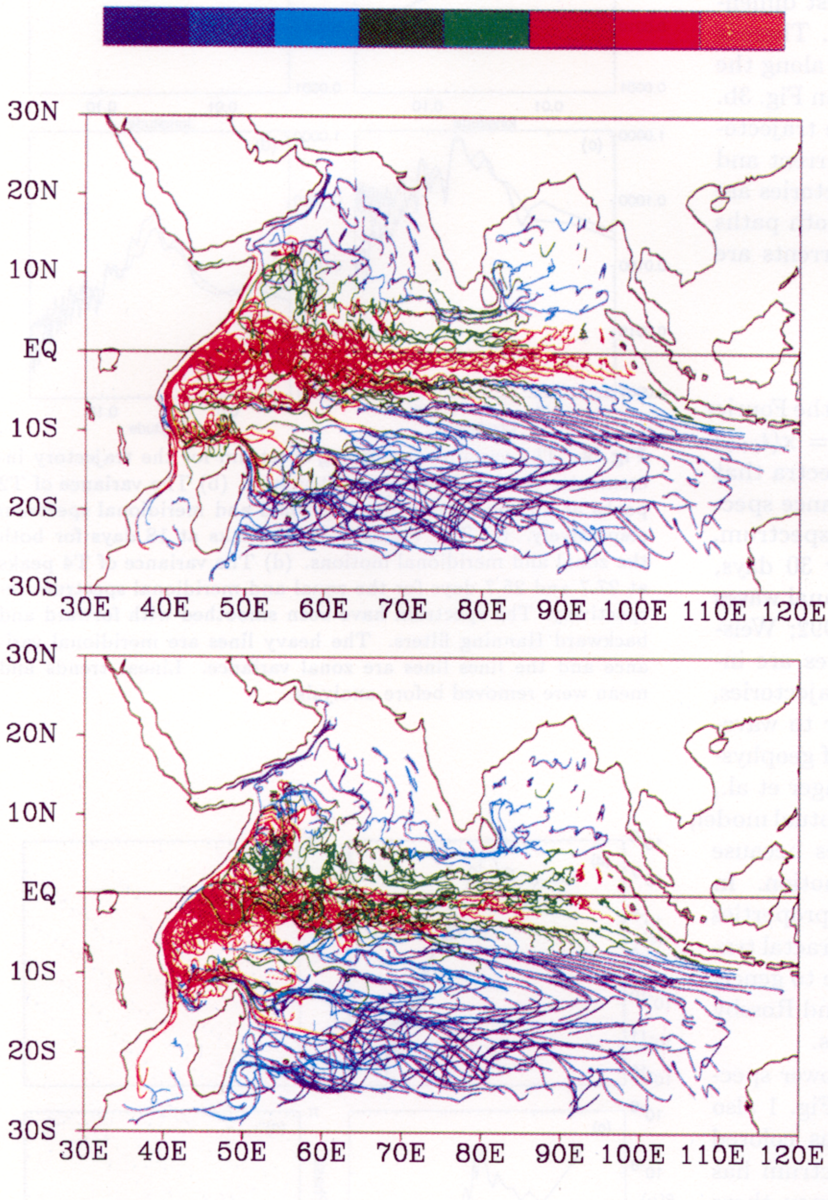

Fig. 3. Trajectories from two model years colored according to their correlation dimension. (a) The first model year. (b) The third model year. The colors range from dark blue ($D = 1.0$) to bright red ($D = 1.3$).

decrease away from the equator. This is unexpected since the generation of fractal trajectories is associated with nonlinear phenomenon and with few exceptions (e.g., Ripa, 1983; Boyd, 1991) studies of equatorial dynamics rest on the assumption of linearity and a variety of measurements (e.g., Tsai et al, 1992; Weisberg, 1987) indicate behavior that is consistent with linear theory.

Within the equatorial wave-guide the highest dimensions are found in the western half of the basin. There is also a meridional extension of high dimension along the coast of Somalia to roughly 10°N, clearly seen in Fig. 3b. This is well outside the wave-guide where the trajectories are influenced by a western boundary current and eddies. Elsewhere the dimension of the trajectories are close to one and the drifters follow very smooth paths through the ocean. In these regions the currents are slow and steady with little seasonal variation.

2.2 Spectral Analysis

Clues to the underlying dynamics are found in the Fourier spectrum of the trajectory increments $\mathbf{v}(t_i) = \mathbf{x}(t_{i+1}) - \mathbf{x}(t_i)$. Analysis of T1 through T4 yields spectra that do not correspond to colored noise. The variance spectra $V(\omega) = \|\omega G(\omega)\|$, where $G(\omega)$ is the raw spectrum, in Fig. 4 are peaked at a period of roughly 30 days, which is the period found in observations of Yanai waves (mixed Rossby-gravity waves) (Tsai et al, 1992; Weisberg, 1987). It therefore appears Yanai waves are involved in the production of some fractal trajectories, perhaps through a mechanism that is similar to wave-induced chaotic mixing in kinematic models of geophysical fluids (Weiss and Knobloch, 1989; Behringer et al., 1991; Pierrehumbert, 1991). Under this conceptual model, Yanai waves can produce erratic trajectories because they are dispersive and induce meridional motion. In contrast, Kelvin waves have neither of these properties (Pedlosky, 1982) and therefore cannot create fractal trajectories. Gravity waves have insufficient scale to generate the large-scale excursions seen in Fig. 2 and Rossby waves do not exist at frequencies near 30 days.

Examination of the phase spectrum and power spectrum $P(\omega) = \|G(\omega)\|^2$ of the trajectories in Fig. 1 also indicate the velocity is not well-described as colored noise. See Fig. 5. Though the energy spectrum has a rough power-law dependence at high frequency, there is a dominant peak in roughly the 25-35 day range. Additionally, the phases for the trajectories do not appear to be random.

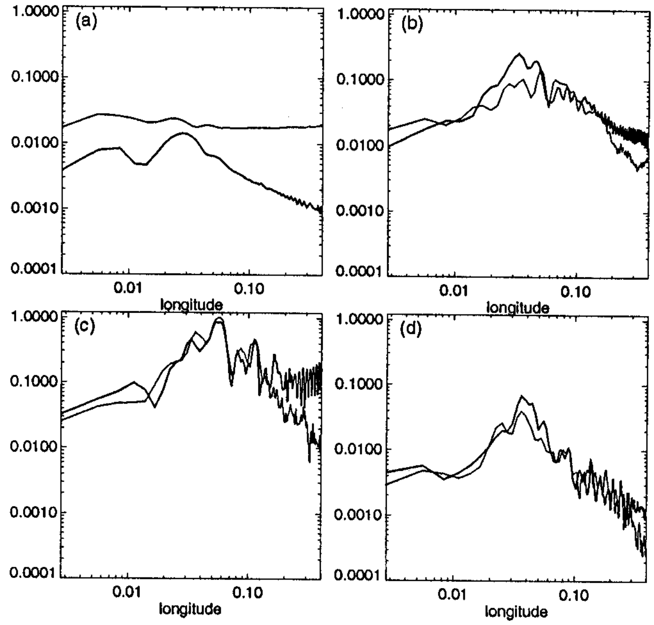

Fig. 4. The variance-preserving spectrum for the trajectory increments. (a) T1 shows broad structure. (b) The variance of T2 peaks at 25 and 30 days for the zonal and meridional spectrum respectively. (c) The variance of T3 peaks at 18 days for both the zonal and meridional motions. (d) The variance of T4 peaks at 27.7 and 25.7 days for the zonal and meridional spectrum respectively. The spectrum have been smoothed with forward and backward Hanning filters. The heavy lines are meridional variance and the lines lines are zonal variance. Linear trends and mean were removed before analysis.

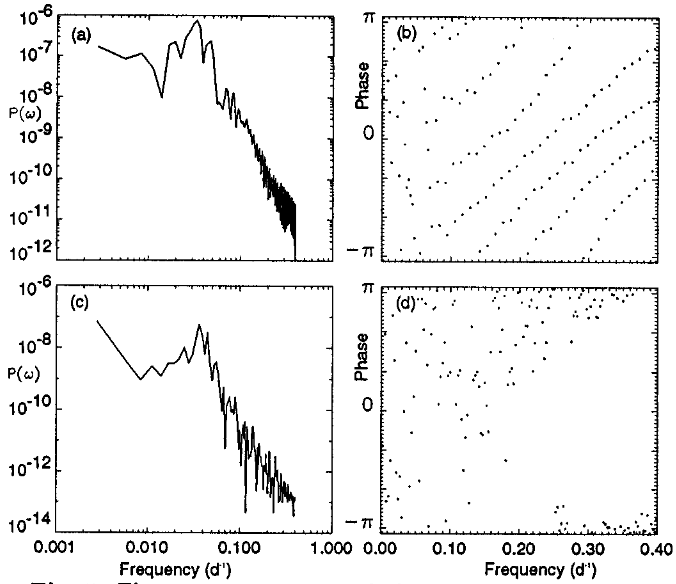

Fig. 5. The energy spectrum and phase spectrum of the meridional velocity for T2 and T4. (a) energy of T2 (b) phase of T2 (c) energy of T4 (d) phase of T4. Linear trends and mean were removed before analysis.

3 Discussion

Lagrangian trajectories from a high-resolution numerical model of the Indian Ocean forced by climatological winds have statistics that are geographically dependent. In particular, their fractal dimension ranges between 1.0

and 1.3, with values greater than about 1.1 occuring almost exclusively within the equatorial wave-guide and along the western boundary. The scaling and correlation dimension of the trajectories were calculated and found to be virtually identical and consistent with each other.

The western domain of the model is a region of intense eddy activity as discussed in Jensen (1990). The eddies produce loops and swirls in the trajectories that result in the development of fractal structure (e.g, T3 in Fig. 1 where $D \simeq 1.22$). Within the wave guide the dynamics are linear and large-scale wave activity is prevalent. The trajectories have statistical properties that differ from those found originally by Osborne et al. (1986). Power and variance spectra of trajectory increments from this region have peaked amplitude at frequencies characteristic of mixed Rossby-gravity waves. It may be that the action of these waves produce fractal trajectories with relatively high dimensions by chaotic advection, as occurs in Hamiltonian models of Rossby waves. This is very different from the stochastic mechanism in the colored-noise model.

Unlike the oceanic or model trajectories, in typical Hamiltonian systems the Lagrangian paths are generally believed to have integer dimension. However, this idealized result is not necessarily attainable in realistic calculations (Benetti et al., 1980). Finding $D < 2$ might be related to the finite sampling time rendering the trajectories too short for accurate determination of dimension. Moreover, dynamical regions in the ocean are not isolated but are directly coupled to other regions with different dynamics. Drifters can wander freely from one region to another. Therefore, erroneous Lagrangian statistics might result not from a measurement time that is too short, but from one that is too long. It is still not possible to discern from this study whether Lagrangian paths in the ocean are best described with Hamiltonian dynamics or with a dissipative dynamical model which itself might be either of low or high dimension. Future studies are planned that will focus on this issue by comparing the above results to statistics of trajectories from different known systems and to real buoy trajectories.

Assuming the fractal trajectories are generated by chaotic advection implies that regions of high dimension are regions of low predictability. This is supported by the fact that the first zero-crossing of the autocorrelation function for the trajectory increments are much smaller for the higher dimension trajectories than for the lower dimension trajectories. For example, the zero-crossing for T1 is 140 days for v_x and 50 days for v_y; for T2 its 110 days for v_x and 10 days for v_y; for T3 its 7 days for both v_x and v_y; for T4 its 10 days for both v_x and v_y. More formal investigations of Eulerian preditability in this model are in progress.

The mechanisms for the production of "fractal" trajectories suggested above are general in nature and do not rely on specific features found in the numerical model used for this study. It is therefore anticipated that similar trajectories will be found in the Atlantic and Pacific equatorial oceans. However, this is a complicated problem due to the presence of equatorial waves, currents, seasonal variations and continental boundaries, each acting and interacting with the others. It may be that some previously unexplored mechanism for generating disorder is operating. One possibility currently under investigation is wave-breaking resulting from the interaction of planetary-scale equatorial waves and the continental boundaries.

The model structure limits which generating mechanisms can be examined. For example, wind forcing frequencies faster than 60 days are eliminated by the month-to-month interpolation of the climatological wind It is likely that in regions where direct wind forcing is dominant, our estimate of dimension is too low. Additionally, the model is exclusively baroclinic, excluding fast barotropic modes from influencing the trajectory.

The limited horizontal resolution of the model imposes an artificial lower limit on the scales forcing the fluid parcel motion. Inclusion of smaller scales (via increased horizontal resolution) may produce an additional scaling region at smaller λ than found in Fig. 2. Alternatively, the chaotic nature of the Lagrangian motion may cause a significant response to these higher frequencies, generating substantially different trajectory statistics. How small-scale structures in the ocean affect the trajectory statistics in the presence of large-scale planetary waves might be studied by examining trajectories from a simple chaotic advection model with additional spatially correlated noise. This remains a topic for future investigation.

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