Nonlin. Processes Geophys., 15, 601–606, 2008
www.nonlin-processes-geophys.net/15/601/2008/

Nonlinear Processes in Geophysics

A new method for abrupt change detection in dynamic structures

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Received: 13 May 2008 – Revised: 30 June 2008 – Accepted: 30 June 2008 – Published: 24 July 2008

Abstract. Based on Detrended Fluctuation Analysis (DFA), we propose a new method – Moving Detrended Fluctuation Analysis (MDFA) – to detect abrupt change in dynamic structures. Application of this technique shows that this method may be of use in detecting time-instants of abrupt change in dynamic structures and we even find that the MDFA results almost do not depend on length of subseries, and are less affected by noise.

1 Introduction

Abrupt change detection in meteorological area is a much studied subject (Alexi et al., 2007; Alley et al., 2003; Broecker, 2003; Ganopolski, et al., 2002; Rahmstorf, 2003; Thomas et al., 1988; Wunsch, 2006). It plays a significant role in improving climate prediction. There are various traditional methods for approaching climate abrupt change detection such as moving t-test, Cramer Method, Yamamoto Method (Yamamoto et al., 1985) and Mann-Kendall test (Mann, 1945; Kendall et al., 1975). These methods have in common that detection results strongly depend on the length of subseries, namely analyzed timescales. Using these traditional approaches, we find the detected time-instants of abrupt change are extremely different for different lengths of subseries. There are two types of abrupt change, one is caused by the change of dynamic equation, and the other is caused by the change of phase or evolution trend (or frequency) of a system but dynamic equation does not change. These traditional methods cannot distinguish the two types of abrupt change. In order to deal with this problem, Pincus (1993, 1995) proposed Approximate Entropy which is generally called ApEn for short and could identify dynamic structure to some extent but still depend on the lengths of subseries. At the same time, there exists a huge drift for the ApEn results which means the actual time-instants of abrupt change don't overlap with the detected one. Making sure of characteristics and time-instants of abrupt change is very important for climate prediction. For that reason, it is essential to provide a new detection method to exactly solve this problem.

Previous studies have shown that a large quality of time series exhibit scaling behaviors (Bunde et al., 2000, 2005; Cao, 1997, 2003; Jan et al., 2007; Liu et al., 2000; Livina et al.,2005; Peter et al., 2000), such as daily temperature records exhibit long-range correlation, which is characterized by an infinite correlation time and linked to power-law behavior of autocorrelation function (Fraedrich et al., 2003), namely:

$$C(s) \sim s^{-\alpha},\ 0<\alpha<1. \qquad (1)$$

A power-law relationship between $C(s)$ and s indicates scaling with an exponent α. Therefore, based on a scaling technique, Detrended Fluctuation Analysis (DFA) (Peng et al., 1993, 1994), we propose a new abrupt change detection method – Moving Detrended Fluctuation Analysis (MDFA). It is mainly used to detect abrupt change in dynamic structures. In this paper, we consider two types of abrupt dynamic structure change:

1. abrupt parameters change in systems;

2. abrupt change caused by external forcing or interactions between systems.

The tests on model time series imply that MDFA overcomes shortcomings of traditional methods and can be used to exactly detect abrupt change in dynamic structures.

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Published by Copernicus Publications on behalf of the European Geosciences Union and the American Geophysical Union.

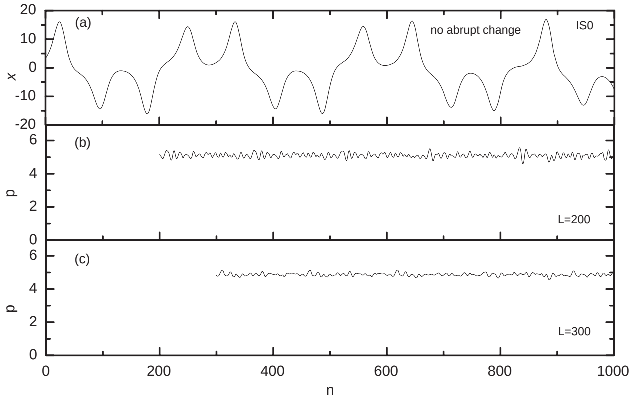

Fig. 1. **(a)** The IS_0 time series $x(i)$ (i=1, 1000) of 1000 units, in which there is no abrupt change in dynamic structure; **(b)** the $MDFA_4$ results of IS_0, the length of subseries is equal to 200; **(c)** same as Fig. 1b, but the length of subseries is equal to 300.

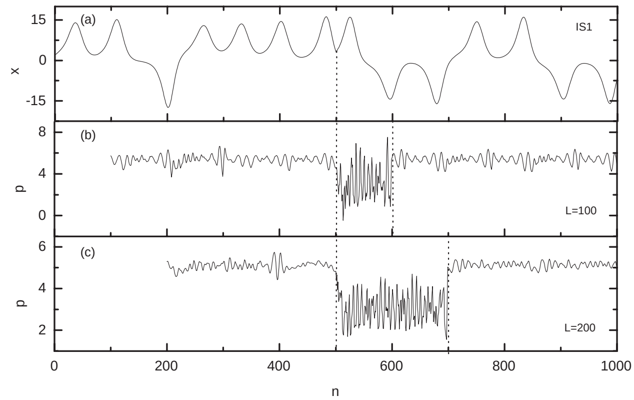

Fig. 2. **(a)** The IS_1 time series $x(i)$ (i=1,1000) of 1000 units, in which there is an abrupt change in dynamic structure when n=501; **(b)** the $MDFA_4$ results of IS_1, the length of subseries is equal to 100; **(c)** same as Fig. 2b, but the length of subseries is equal to 200.

This paper is organized as follows: Sect. 2 describes the abrupt change detection methodMDFA and the data in tests, and then we provide the results and discussion in Sect. 3. Finally, our conclusions are presented.

2 Method and data

Estimating the autocorrelation function $C(s)$ from empirical data is limited to rather small lag s and is affected by observational noise and nonstationarities such as trends (Maraun et al., 2004). Therefore, in this paper, we employ DFA, a scaling analysis method that can deal with seemingly non-stationary time series, and which provides a simple quantitative parameter – scaling exponent – to describe power law in time series (Peng et al., 1993, 1994). The relationships between scaling exponents α and p are (Bunde et al., 2000, 2005; Maraun et al., 2004):

$$\alpha=2(1-p). \quad (2)$$

Different orders n of DFA (DFA_1, DFA_2 etc.) differ in the order of the polynomial used in DFA procedure (Monetti et al., 2003). Accordingly, different order n of MDFA procedure can be marked by $MDFA_n$. In order to detect abrupt change in dynamic structures which exist in time series, we first choose a subseries to calculate a scaling exponent by DFA, and then move subseries gradually without changing the length of subseries, repeat this operation until the end of the original series. If there is an abrupt change in dynamic structure, the scaling exponent will have a very sharp change in the vicinity of abrupt change. Whereas, if there is no abrupt change in dynamic structure, the scaling exponent will have a relative tiny change, and this change mainly caused by an insufficiency of samples size. Based on these characteristics, we can easily identify time-instants of abrupt change in dynamic structures. The data in tests come from chaos model, such as classical Lorenz model (Lorenz et al., 1963) or Chen model (Chen et al., 1999), and the data display scaling character. In this paper, all the lengths of model time series are 1000 units.

3 Results and discussion

Figure 1a shows a model time series IS_0, which is produced by Lorenz system when Rayleigh parameter R=28.0, and there is no abrupt dynamic change in IS_0. Figure 1b and c shows the $MDFA_4$ results for the model time series used in Fig. 1a. When the length of subseries is equal to 200, the maximum and minimum of scaling exponents is respectively 5.57343, 4.60895 and the variation of scaling exponents is about 0.96448. When the length increases to 300, the maximum and minimum of scaling exponents are respectively 5.14896, 4.55396 and the variation approximately decreases to 0.595. We use $MDFA_4$ to detect other model time series which have no abrupt dynamic change, and we find that the results are similar to that of IS_0. The variation of scaling exponents is relatively small, which is a common character for those model time series without abrupt dynamic change. Meanwhile, we find that the variation of scaling exponents decreases with the increase of length of subseries. Consequently, for those time series without abrupt dynamic change, it is obvious that fluctuation of scaling exponents induced by moving subseries mainly due to the shortage of sample size.

To illustrate parameter change which induces abrupt change in dynamic structure, we use the model time series IS_1 which is formed by the variable x in Lorenz system (see Fig. 2a). The previous half in IS_1 are produced when Rayleigh parameter R=29.0, the others are produced when R=28.0. Obviously, there is an abrupt parameter change in IS_1 when n=501.

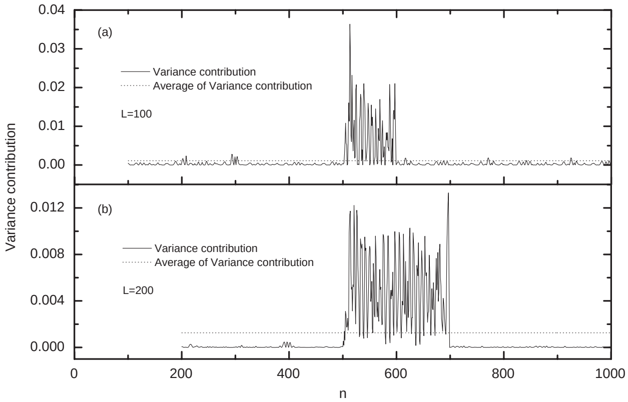

Fig. 3. Variance contributions of scaling exponents. **(a)** Variance contributions for detection results of IS_1 by using $MDFA_4$, the length of subseries is equal to 100; **(b)** same as Fig. 3a, but the length of subseries is equal to 200.

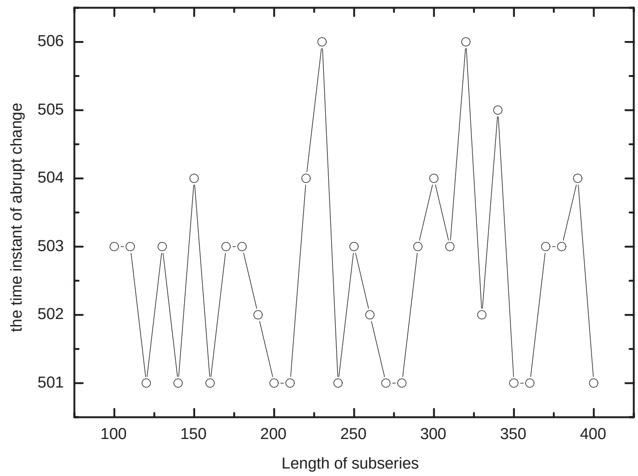

Fig. 4. The time-instants of abrupt dynamic change as a function of length of subseries for IS_1.

Figure 2b and c shows the $MDFA_4$ results for the model time series IS_1, respectively. According to the difference of fluctuation amplitude of scaling exponents, We can easily find from Fig. 2b that the evolution curve of scaling exponents can be roughly divided into three parts:

1. (100,500);
2. (501,600);
3. (601,1000).

In the first and third parts, the fluctuation of scaling exponents is stable, additionally fluctuation amplitude is relatively small and the variation is about 2.96393. As said before, the relatively small fluctuation amplitude mainly dues to the shortage of sample size. But in the second part, the fluctuation amplitude of scaling exponents increase apparently and the variation is about 7.99496. The reason is that MDFA is very sensitive to the data from different system. In other words, if subseries are comprised of data coming from different dynamic systems, the variation of scaling exponents apparently greater than that of subseries, which are comprised of data coming from identical dynamic system. Figure 2b and c shows that $MDFA_4$ can be used to detect the abrupt dynamic structural change in the model time series IS_1.

To exactly give time-instants of abrupt dynamic change from the $MDFA_4$ results, we need present a quantitative measure. In view of the sensitivity of MDFA to the data coming from different dynamic systems, in this paper, we estimate the time-instants of abrupt change quantitatively from the MDFA results according to the variance contribution of scaling exponents. It should be noted that the average of scaling exponents in variance contribution procedure is calculated by scaling exponents, the variation of which is relatively smaller. For example, when the length of subseries is 100, the average of scaling exponents for the model time series IS_1 is calculated by the scaling exponents in the first and third parts in Fig. 2b. Figure 3a and b presents the results of variance contribution procedure of scaling exponents, according to Fig. 2b and c, respectively. We find that the plots of variance contribution can distinguish normal fluctuation amplitudes from abnormal ones. The normal one is mainly caused by the shortage of sample size, and the abnormal one is primarily caused by the sensitivity of MDFA to the data from different system. We define the time-instant of abrupt change is the point when the first variance contribution is above the average of variance contribution. Based on this definition, we find that all the time-instants of abrupt dynamic change in IS_1 are n=503 when the lengths of subseries are respectively 100 and 200. Then we use $MDFA_4$ to analysis IS_1 for other different lengths of subseries, and the results have been shown in Fig. 4. It can be seen from Fig. 4 that the time-instant of abrupt change range from 501 to 506, which are all very close to the actual time-instant of abrupt change (n=501). The MDFA results indicate that MDFA can be used to effectively detect abrupt dynamic change, and hardly depend on the length of subseries. We can get similar results by using other orders of MDFA.

In this paper, we test the performance of traditional abrupt change detection methods including Moving t-test, Cramer method, Mann-Kendall test, Yamamoto method and ApEn, and compare them with the corresponding MDFA results. The detection results of Moving t-test, Cramer method and Mann-Kendall test are summarized in Table 1, and it is very easy to find that the results of traditional methods highly depend on lengths of subseries. For example, when the length of subseries is 100, the numbers of abrupt change by using Moving t-test, Cramer method and Mann-Kendall test are 8, 5, and 5 respectively. But when the length increases to 200, the number of abrupt change gained from the three traditional methods becomes 3, 1 and 3 respectively. With the increase

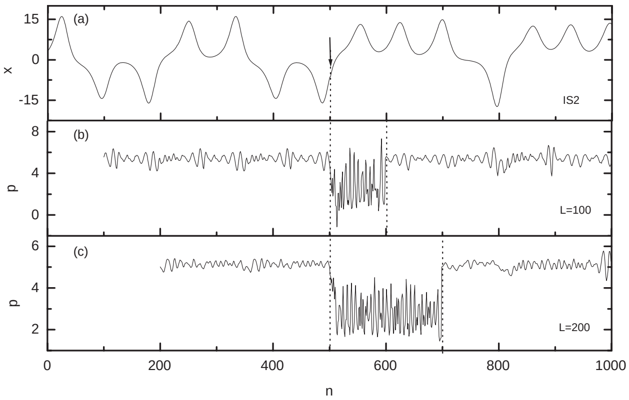

Fig. 5. Same as Fig. 2, but the time series is IS_2.

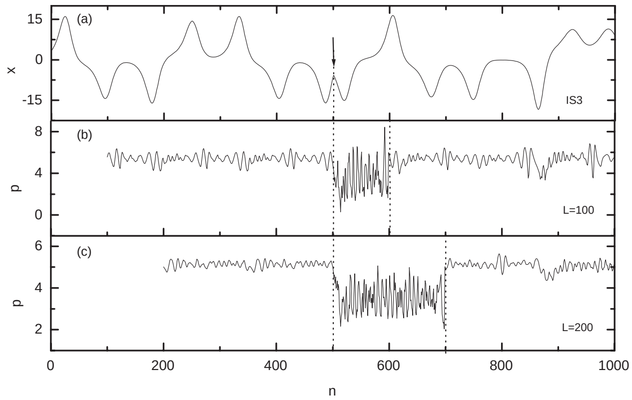

Fig. 6. Same as Fig. 2, but the time series is IS_3.

Table 1. The number of abrupt change in IS_1 for different lengths of subseries by using traditional methods.

Traditional Method	Length of subseries 100	200	300
Moving t-test	8	3	1
Cramer Method	5	1	0
Mann-Kendall	5	3	2

of length of subseries, there is a decreasing trend in the numbers of abrupt change. The Yamamoto method cannot detect any abrupt change in IS_1. The ApEn results indicate that the time-instant of abrupt change depends on the length of subseries, for example, the detected time-instants of abrupt change are n=181, 392, 487, 530 for the length L=100, 200, 300, 400, respectively. It is obvious that the performance of MDFA is better than that of traditional methods.

Model time series IS_2 is composed of two parts: the classical Lorenz model (Lorenz et al., 1963) and the periodic forcing Lorenz model, which is as follows:

$$\frac{dx}{dt}=-\sigma x+\sigma y+A\cos(wt)$$
$$\frac{dy}{dt}=Rx-y-xz$$
$$\frac{dz}{dt}=xy-bz \qquad (3)$$

A is the forcing strength, we get 0.005 here, $A\cos(wt)$ is the forcing term. W is the frequency of periodic forcing, we get 0.02 here. Parameters σ, b are 10.0 and 8/3, respectively. R is Rayleigh number, which is equal to 28.0. Figure 5a shows the evolution curve of the model time series IS_2, and the first 500 data are produced by the variable x in classic Lorenz equations, the others are produced by the variable x in Eqs. (1). Figure 5b and c show the $MDFA_4$ results for different lengths of subseries. Based on variance contribution procedure (Figs. omit), we find that the time-instant of abrupt change is n=504, which is very close to the actual time-instant of abrupt change. The model time series IS_2 is very smooth, and so it is usually difficult to find out the time-instant of abrupt dynamic change. But we can easily and exactly detect the time-instant of abrupt change through $MDFA_4$.

Climate system includes various kinds of subsystems. There exists coupling between different climate systems, sometimes it is strong and sometimes it is weak. A coupling can be ignored when its coupling strength is weak, however, it cannot be ignored when its coupling strength is strong. The strong interactions between subsystems may induce the evolution of climate system to an exceptional state, such as floods, drought, extreme high temperature etc. We now consider a simple binary model and its strong interactions between subsystems have caused abrupt dynamic change. This model is composed of Lorenz model and Chen system (Lorenz et al., 1963; Chen et al., 1999), the coupling binary model can be written as:

$$\frac{dx_1}{dt}=-\sigma x_1+\sigma y_1+E(x_1-x_2)$$
$$\frac{dy_1}{dt}=r_1x_1-y_1-x_1z_1$$
$$\frac{dz_1}{dt}=x_1y_1-b_1z_1$$
$$\frac{dx_2}{dt}=-ax_2+ay_2+w_2-E(x_1-x_2)$$
$$\frac{dy_2}{dt}=dx_2+cy_2-x_2z_2$$
$$\frac{dz_2}{dt}=x_2y_2-b_2z_2$$
$$\frac{dw}{dt}=y_2z_2-r_2w \qquad (4)$$

x_1, y_1, z_1 are three variables of Lorenz model, respectively, and x_2, y_2, z_2, w are four variables of Chen system. $E(x_1-x_2)$ is the coupling term. The two subsystems can achieve interactions through coupling between variable x_1 and x_2. E is the coupling strength, which represents the strength of interactions between two subsystems. When $E \rightarrow 0$, there is no interactions. But the interactions cannot be ignored with the increase of coupling strength E, and the evolution of subsystem will depart the unperturbed state, which directly induce abrupt change in dynamic structure. Other parameters in Lorenz model are same as that in Eq. (1). In Chen system, parameter a is 35, d is 7, c is 12, b_2 is 3, and r_2 is 0.08, respectively.

In the model time series IS_3, the first 500 data have been created by the variable x in the classical Lorenz model, and the second 500 data have been created by the variable x_1 in the coupling model (Eq. 2). The coupling strength E between the two subsystems is 0.001 in this test. The evolution curve of the model time series IS_3 has been shown in Fig. 6a. Similar to IS_1, We can easily find that the evolution curve of scaling exponents can be roughly divided into three different parts based on different fluctuation amplitudes. Then we use variance contribution procedure to analyze scaling exponents in Fig. 6b and c, and find that the time-instants of abrupt change are respectively n=503 and 504. And then we studied the effect of noise on MDFA because noise is inevitable in observational data. Figure 7 shows the MDFA results for IS_1 under different Signal Noise Ratio (SNR), and we find that noise has an impact on the MDFA results, especially for strong noises, but MDFA has a perfect capability to resist the effect of noise. The MDFA results of analogous tests on noise for IS_2 and IS_3 are similar to that for IS_1.

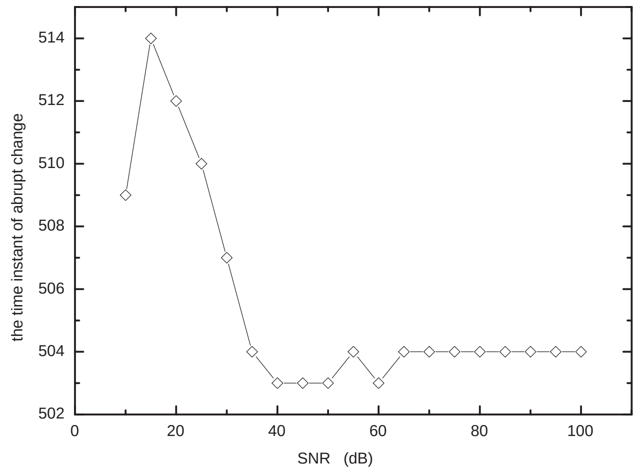

Fig. 7. The MDFA results of IS_1 under different SNR, the length of subseries is equal to 200.

4 Conclusions

We present a new method-MDFA for detection of abrupt change in dynamic structures. The tests on different model time series indicate that this new method can be able to effectively detect abrupt dynamic change. What's more, the MDFA results are almost independent of length of subseries and have a perfect capability to resist the effect of noise. The MDFA results are observably better than that of tradition methods because detection results of traditional methods evidently depend on lengths of subseries analyzed. The MDFA provides a reliable approach to estimate time-instants of abrupt dynamic change and overcomes the drawback of traditional ones which cannot effectively detect abrupt change in dynamic structures. Based on this, this new method must be applied in the future widely.

Acknowledgements. The authors thanks anonymous reviewers and editors for beneficial and helpful suggestions for this manuscript. This work was jointly supported by the National Natural Sciences Foundation of China (Grant No. 40325015), the National Key program development for Basic Research (Grant No. 2006CB400503).

Edited by: J. Kurths
Reviewed by: One anonymous referee

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