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Nonlinear Processes in Geophysics

Brief communication "On one mechanism of low frequency variability of the Antarctic Circumpolar Current"

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Abstract. In this paper we present a simple analytical model for low frequency and large scale variability of the Antarctic Circumpolar Current (ACC). The physical mechanism of the variability is related to temporal and spatial variations of the cyclonic mean flow (ACC) due to circularly propagating nonlinear barotropic Rossby wave trains. It is shown that the Rossby wave train is a fundamental mode, trapped between the major fronts in the ACC. The Rossby waves are predicted to rotate with a particular angular velocity that depends on the magnitude and width of the mean current. The spatial structure of the rotating pattern, including its zonal wave number, is defined by the specific form of the stream function-vorticity relation. The similarity between the simulated patterns and the Antarctic Circumpolar Wave (ACW) is highlighted. The model can predict the observed sequence of warm and cold patches in the ACW as well as its zonal number.

1 Introduction

The Antarctic Circumpolar current (ACC) is a strong flow transporting about 130 Sverdrups. It encircles the Antarctica thus connecting waters between the Atlantic, Pacific, and Indian Oceans. Available observations suggest that the ACC contains a series of sharp temperature and density fronts, each separating two distinct water masses. Two major circular fronts in the ACC are the Subantarctic Front (SAF) and the Polar Front (PF), whose mean paths are now well defined (Gille, 1994). The ACC flow varies with time and its variability can be divided into the short-timescale variability due to the local effect of mesoscale eddies (e.g. Sarukhanian, 1985) and lower-frequency variability at greater spatial extent. In this paper, we will focus on effects related to lower-frequency variability, which has the potential to influence larger-scale ocean flow and climate variability. An example of such low-frequency variability is the Antarctic Circumpolar Wave (ACW) discovered by White and Peterson (1996) and Jacobs and Mitchell (1996). The ACW consists of anomalies in sea-surface temperature, sea-level pressure, and sea-ice extent (Connolley, 2003) that propagate eastward around the Southern Ocean because of advection by the ACC.

In this paper, we present a theoretical model to a mechanism leading to variability of the ACC due to the Rossby wave trains propagating between the SAF and PF, which are conceptualized as boundaries of the waveguide that provide meridional trapping for the Rossby waves. Flow in these Rossby wave trains slowly varies with time thus contributing to low frequency variability of the ACC, similar to that observed in the ACW. The patterns in the ACW were initially identified as zonal wave number 2 (ACW2). They circle the globe in about 8–9 yr, thus 4 yr is the apparent period at any location (White and Peterson, 1996), (Jacobs and Mitchell, 1996). According to calculations by Cai et al. (1999), the ACW has zonal number 3 (ACW3). Recently, Venegas (2003) reported that the inter-annual band during the period 1980–2000 is the linear combination of two signals, namely, the ACW2 and the ACW3 with different temporal and spatial characteristics. The physics of the ACW, in particular the role of coupling in the ocean-atmosphere system, has been actively debated. The initiation of the ACW may be the result of atmospheric teleconnections related to the El Nino-Southern Oscillation or arises from, or is at least maintained by, atmosphere-ocean coupling within the Southern Ocean. Other authors argue that the ACW is a passive ocean response to atmospheric forcing, and not a true coupled mode. These mechanisms are reviewed in Rintoul et al. (2001). In this paper we will neither discuss the mechanisms for initiation of the ACW, nor the ocean-atmosphere coupling or forcing

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issues. Our aim is to show that the Rossby wave train between the SAF and PF is a fundamental mode of motion in the Southern ocean, and to present a simple mathematical model for this mode. Indeed, only the fundamental mode of motion can be sustained for long periods of time under varying ambient conditions and complicated atmosphere-ocean interactions. A model for this fundamental mode, including prediction that zonal number for the patterns is likely to be 2 or 3, with intermittent warm and cold patches, is the major result of the paper.

2 Simplified theory

We consider the two-dimensional flow of a purely barotropic fluid, which is rotated with an angular velocity c around a geographical pole in a circular channel of the width L . We use the gamma plane approximation, which means that the exact expression for the Coriolis force is expanded about a pole. This approach differs from the widely used beta plane approximation where expansions are about some mid-latitude location. We introduce the stream function in the usual way $u = \psi_\xi$, $v\,\xi = -\psi_\phi$, where u, v are velocities in the eastward and the northward directions; ξ is dimensionless radial distance to the geographical pole scaled with the channel width L; ϕ is the longitude. Our idea is that a circular channel created by the SAF and PF acts like a waveguide with non-penetratable boundaries. The dimensionless equation for the stream function ψ in this case is the following e.g. Derzho and Afanasyev (2008),

$$\nabla^2\psi - \frac{\varepsilon^2}{Ro}\frac{\xi^2}{2} = F(\psi - c\frac{\xi^2}{2}),\ \varepsilon = L/R_E \ll 1, \tag{1}$$

where F is an arbitrary function to be determined from the relation between the potential vorticity and the stream function at some cross sections of the flow. As it follows from the latter inequality, we consider the length scale of the flow to be much smaller that the radius of the Earth R_E. $Ro = U/2\Omega L$ is the Rossby number, Ω denotes the rotation rate of the Earth, U is a typical circumferential velocity. We also use the hydrostatic and rigid lid approximations. The validity of these approximations for the problem was discussed by Derzho and de Young (2011). Classical papers like Stern (1975) assumed that F is linear. We will assume that the potential vorticity-stream function relation F has a weakly nonlinear functional form thus extending the classical approach.

$$F = -\lambda(\psi - c\frac{\xi^2}{2}) + \sigma^2\ f(\psi - c\frac{\xi^2}{2}),\ \ \sigma \ll 1, \tag{2}$$

where λ is a constant and f determines the functional nonlinearity in the vorticity-stream function relation. That weakly nonlinear relation was experimentally confirmed by Derzho and Afanasyev (2008). Here we focus on the solutions of Eq. (1) whose radial variations appear at much shorter scales compared to the angular one $\psi = \psi(\xi, \Phi)$, $\Phi = \sigma\phi$. We seek asymptotic solutions of Eqs. (1, 2) in the form of power series in the small parameter σ^2. We look for solutions of Eqs. (1, 2) in a circular channel with inner and outer rigid boundaries located at $\xi = R_1$ and $\xi = R_2$, respectively. As shown by Derzho and de Young (2011), the regular zeroth order solution of Eqs. (1, 2) satisfying the boundary conditions has the form,

$$\psi^{(0)} = (\frac{\xi^2}{2} - \frac{2}{\lambda})(\frac{\varepsilon^2}{\lambda\ Ro} + c) + A(\Phi)\ W(\xi), \tag{3}$$

where the function A in Eq. (3) is to be determined from the solvability condition to the first approximation. The constant λ and function $W(\xi)$ can be determined for given values R_1 and R_2 as shown in Derzho and de Young (2011). The first term in Eq. (3) has physical meaning of an ambient mean current, which is the ACC in our study. The second term denotes a disturbance superimposed on that current, it corresponds to the ACW-like pattern in our application. It can be shown from Eq. (3) that the mean current is approximated as the current with a constant angular velocity. The non-dimensional expression for that velocity c_m and its dimensional counterpart, $c_{m,\mathrm{dim}}$, are the following,

$$c_m = \frac{\varepsilon^2}{Ro\ \lambda} + c,\ c_{m,\mathrm{dim}} = \frac{L^2}{R_E^2}\frac{2\Omega}{\lambda} + c_\mathrm{dim}. \tag{4}$$

Taking L from observations, then calculating non-dimensional distances R_1 and R_2 from the pole to the SAF and PF, and calculating λ according to the procedure described in Derzho and de Young (2011), Eq. (4) determines the dimensional angular velocity for the rotating Rossby wave pattern if the mean current is prescribed using the data from observations. The next step is to resolve the spatial structure of the Rossby wave pattern. Equation (1) is to be solved along with the boundary conditions applied to the first order stream function. However, to determine the spatial structure of the zeroth order solution, we only need to consider the solvability condition to the first order stream function. Re-writing Eq. (1) in a self-adjoint form and applying the solvability condition yields the equation for the amplitude function $A(\Phi)$ similar to that in Derzho and de Young (2011). Equation (3) and the expression for $A(\Phi)$ define the spatial pattern for the stream function in the leading order. As it was shown in laboratory experiments (Derzho and Afanasyev, 2008) specially designed for modelling of polar flows, the vorticity – stream function relation is weakly nonlinear. A straightforward extension beyond a linear relationship usually used by many authors, e.g. (Stern, 1975), is a quadratic polynomial for f. This choice of f reduces the solvability condition to the Korteveg-de Vries type equation for the amplitude of Rossby wave,

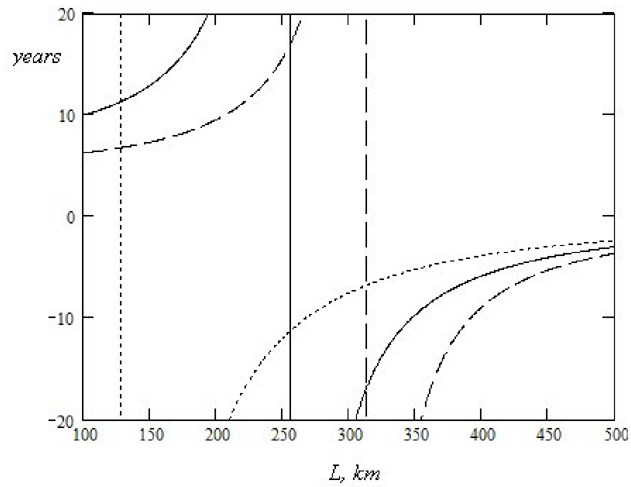

Fig. 1. Period of rotation of the Rossby wave train in years versus the waveguide width (mean distance between the SAF and PF), in km for different magnitudes of the mean current: $2\,\mathrm{cm\,s^{-1}}$ – dots, $8\,\mathrm{cm\,s^{-1}}$ – solid line, $12\,\mathrm{cm\,s^{-1}}$ – dash line. The negative period indicates that the Rossby wave train propagates against the mean current

$$f(B) = s_1\ B + s_2 B^2, A_{\Phi\Phi}\ = I_0 + A\ \ I_1 + A^2\ I_2, \tag{5}$$

where I_0, I_1, and I_2 are constants that depend on s_1, s_2, c, and c_m.

Numerical analysis of Eq. (5) along with the condition of zero circulation leads to the expression for the Rossby wave amplitude as function of longitude. Analytical results for the case with zero mean flow were presented in Derzho and de Young (2011), these results are extended here for the more realistic case with ambient mean current. Equation (5) has solutions in the form of cnoidal waves with the wavelength $\frac{2\pi}{N}$ (for the original angular coordinate ϕ, $N = 1, 2, ...$ where N is the zonal number. Our specific choice of N corresponds to a $2N$-pole. Similarly to Derzho and de Young (2011), it can be shown that the problem has a unique solution once s_1, s_2, and the magnitude of the ambient shear are prescribed. That is the wave amplitude and spatial structure are completely and uniquely defined by the specific form of the vorticity-stream function relation and the magnitude and the width of the ambient current.

3 Application to the ACC conditions

Here we present our calculations for the special case of a vortical pattern rotating around the Antarctic aimed at explaining a mechanism for slow frequency and large scale variability of the ACC. Our solution for a vortical pattern in a polar region consists of two parts, the first part is a Rossby wave while the second part is a current. Our physical assumption is that the Subantarctic Front (SAF) and the Polar Front (PF), which are reasonably well-defined narrow jets (about 40 km wide) create a wave guide for a Rossby wave train. Gille (1994) analyzed GEOSAT altimetry data and presented the mean paths of the SAF and PF jets. Although topography affects the tracks of these paths, they remain quite close together around the Antarctic continent. According to Fig. 9 from Gille (1994) the distance between mean SAF and PF paths varies from about 100 km to more than 500 km in some parts of the South Indian Ocean. However, since these distances are much smaller than the distance from either SAF and PF to the South pole, the SAF and PF paths can be approximated as two circumferences. As the SAF and PF separate distinct water masses, we conceptualize them as rigid boundaries, which create a circular channel. In our calculations, we choose the internal circumference at which the radial velocity is zero to be located at 3400 km. First, it is interesting to note that according to Eq. (4) the angular velocity of rotation of the vortical structure is independent of the zonal number. This conclusion is supported by the frequency domain decomposition of the interannual variability in the Southern Ocean performed by Venegas (2003), where variability was detected with time scale of 5 yr for the zonal mode 2 and 3.3 yr, for the zonal mode 3, respectively. The resulting period of rotation for the whole patterns is then 10 yr, same for both zonal modes 2 and 3. As can be seen from Eq. (4), the period of rotation of the vortical pattern (Rossby wave train) is independent of the specific coefficients σ, s_1, s_2 in the stream function-vorticity relation Eq. (2). It depends,however, on the width and the magnitude of the ambient current. In Fig. 1 we show the period of rotation of the vortical pattern as a function of the distance between the SAF and PF for various magnitudes of the mean current, namely $2\,\mathrm{cm\,s^{-1}}$, $8\,\mathrm{cm\,s^{-1}}$ and $12\,\mathrm{cm\,s^{-1}}$. It is seen that the Rossby waves propagate against the current if the waveguide between the SAF and PF is sufficiently wide. In this case, the vortical patterns propagate around the Antarctica in 3–4 yr, and this period only weakly depends on the magnitude of the mean current. As the distance between the SAF and PF becomes shorter, the period of rotation increases, and reaches infinity (patters do not rotate at all) at some width of the waveguide. When the distance between the SAF and PF further decreases, the pattern sense of rotation changes, and the pattern rotates in the same direction as the mean current does. The width at which the rotation of the pattern change direction is strongly affected by the magnitude of the mean current. Venegas (2003), among others, reported that the velocity of propagation of anomalies of the sea surface temperature observed in the ACC, is about $8\,\mathrm{cm\,s^{-1}}$. In our theory, no mean circulation due to the Rossby waves themselves is assumed, so it is reasonable to identify this velocity with the average velocity of ambient current. A magnitude of the mean current of $8\,\mathrm{cm\,s^{-1}}$ at the inner boundary, according to Eq. (4), leads to a period of 8.6 yr for rotation of the Rossby wave pattern, when the length scale (waveguide width) is set at $L = 360$ km. This period is close to that observed for the

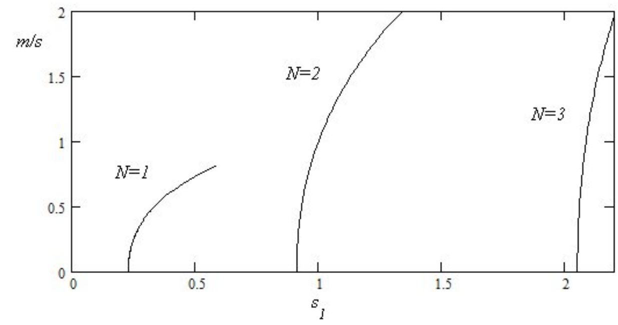

Fig. 2. Maximum dimensional circumferential velocity at the outer circumference (3760 km) versus s_1 for mode numbers $N = 1, 2, 3$

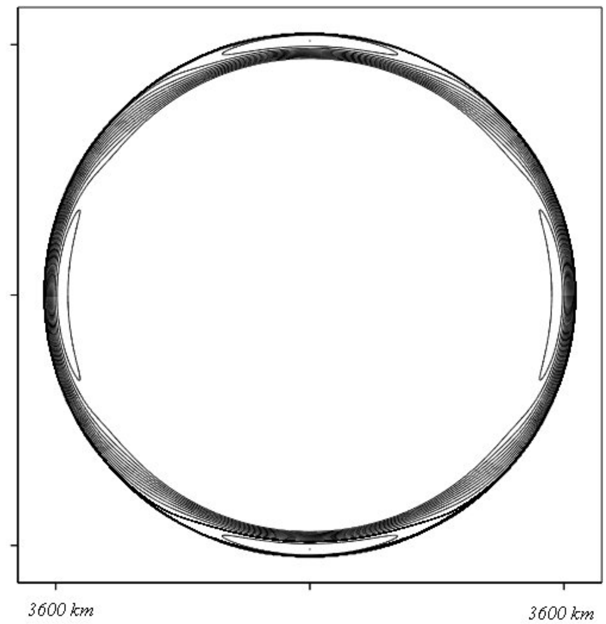

Fig. 3. The streamline pattern for the quadrupole case ($N = 2$); $\sigma = 0.3$, $s_1 = 0.912$

variability of the ACC reported in White and Peterson (1996) who suggested periods of 8–9 yr. As the wave guide is quite narrow, so there is no much difference between the mean current magnitudes at the inner and the outer boundaries of the waveguide according to Eq. (3). Our choice for the magnitude of the ambient current and the length scale leads to a flow rate of 121 Sv if we assume that that the average depth of the oceans is 4 km. This value is close to 130 Sv, the flow rate in the ACC, reported by many authors. It can be shown (Derzho and de Young, 2011) that solutions to our problem can only exist if $s_2 < 0$. Here we examine the case $s_2 = -1$ and set $\sigma = 0.3$; these parameters affect the magnitude of the velocity field in the wave for given s_1. We will show below that this choice of parameters lead to reasonable agreement with observations. With all these parameters are set, it is straightforward to analytically determine possible values of s_1 for which stationary rotating vortex patterns exist. Basically, in the present model, we aggregate many physical effects into a couple of coefficients in the vorticity-stream function relation. The advantage of such approach lies in simplicity as separate parameterisation of many complicated processes, in particular, the ocean-atmosphere interactions, leads to accumulation of inaccuracies associated with each parameterisation.

In Fig. 2, we present the maximum dimensional circumferential velocity at the outer circumference versus s_1 for various zonal mode numbers $N = 1, 2, 3$. The maximal radial velocities are not shown as they are much smaller than the circumferential ones. It can be seen that regions of existence for different zonal modes are separated. There are intervals where the only mode 1 or mode 2 only can exist. Regions of existence for modes 2 and 3 can overlap. But if mode 3 is excited, it is unlikely that the mode 2 exists at the same value of s_1 as the mode 2 pattern should produce quite large flow velocities.

Again, we recall that the leading order stream function patterns can be found from Eqs. (3), (4), (5). These patterns are shown in Fig. 3 for the case of quadrupole ($N = 2$); $s_1 = 0.912$. The pattern is quite symmetric and the actual cnoidal Rossby wave train is close to a sinusoidal packet. However, this does not mean that the linear theory can predict properties of such flow, actually our theory predicts that there is a unique amplitude that satisfies our mathematical formulation for the prescribed set of parameters. Linear theories are unable to predict an amplitude of the wave and consequently do not predict the magnitude of the velocity field in it. For the mentioned set of parameters, we calculated that the maximum of the circumferential velocity due to the waves is $15\,\mathrm{cm\,s^{-1}}$, while the magnitude of the the radial velocity is $1\,\mathrm{cm\,s^{-1}}$. We also note that the variation of σ does not qualitatively change the results. For example, when $\sigma = 0.3$ and $s_1 = 0.912$, the predicted Rossby wave pattern has exactly the same velocity field as for $\sigma = 0.1$ and $s_1 = 8.208$.

The most important feature of the pattern shown in Fig. 3 is that the model predicts the existence of the closed streamlines in the flow, which in nature can be treated as patches of trapped fluid. Two regions of closed streamlines are attached to the inner boundary (PF), which separates cold Antarctic waters and the ACC waters, these two patches are supposed to be cold bodies of water. Two other regions of closed streamlines are attached to the outer boundary (SAF),which separates the ACC and warmer waters from the North. These two patches are expected to be filled with warm waters captured from the outside the ACC. The ACW also has two trapped bodies of cold water and two of warm water, and the observed sequence of warm-cold-warm-cold is the same as predicted by our model.

4 Conclusions

Here we modeled the variability of the ACC due to nonlinear Rossby wave trains with zero circulation. The train is predicted to rotate around the Antarctica. The angular velocity of rotation of the vortical Rossby wave pattern is not arbitrary, but is predicted to be at a specific rate, which is independent of the zonal number but rather depends on the distance between the SAF and PF, and the strength of the mean current. The pattern propagates against the ACC for large distances between the SAF and PF and collateral to the ACC when this distance is short. Once the vorticity-stream function relation is prescribed, our model predicts that there is a unique nonlinear vortex pattern for each zonal mode number. Various modes are allowed to exist, however they have reasonable amplitudes (those leading to the observed velocities) in different range of parameters in the stream function-vorticity relation, perhaps indicating that they are driven by different physical mechanisms. This conclusion is consistent with observations reported by Venegas (2003). Reasonable values for the distance between the SAF and PF and the magnitude of the average mean current lead to reasonable agreement for the period of rotation and the velocity field characteristics. The theory also predicts that the Rossby wave patterns include patches of trapped fluid that could carry fluid from the inner (colder) and outer (warmer) boundaries of the ACC thus explaining the sequence of the warm and cool patches carried by the ACW. A more detailed modeling of the ACC and ACW would require a spatially explicit numerical model, which is beyond the scope of the present study. We also believe that the detailed account of the spatial features, e.g. bathymetry, would not qualitatively change the main idea of the proposed mechanism for the low frequency variability of the ACC.

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References

Cai, W., Baines, P. G., and Gordon, H. B.: Southern mid- to high-latitude variability, a zonal wavenumber-3 pattern, and the Antarctic Circumpolar Wave in the CSIRO coupled model, J. Climate, 12, 3087–3104, 1999.

Connolley, W. M.: Long term variation of the antarctic circumpolar wave, J. Geophys. Res., 108 (C4), 8076–8098, 2003.

Derzho, O. G. and Afanasyev, Y. D.: Rotating dipolar gyres on a gamma-plane, Phys. Fluids, 20, 036603, 2008.

Derzho, O.G. and de Young, B.: Rotating nonlinear vortical patterns on a gamma plane, Stud. Appl. Math.,126, 186–201, 2011.

Gille, S. T.: Mean sea surface height of the antarctic circumpolar current from geosat data: methods and application, J. Geophys. Res., 99(C9), 18255–18273, 1994.

Jacobs, G. A. and Mitchell, J. L.: Ocean circulation variations associated with the antarctic circumpolar wave, Geophys. Res. Lett., 23, 2947–2950, 1996.

Rintoul, S. R., Hughes, C. W., and Olbers, D.: The Antarctic Circumpolar Current system, in: Ocean Circulation and Climate, edited by: Siedler, G., Church, J., and Gould, J., International Geophysical Series, Academy Press, New York, 271–302, 2001.

Sarukhanian, E. I.: Structure and Variability of the Antarctic Circumpolar Current, Oxonian Press, New Delhi, 1985.

Stern, M.: Minimal properties of planetary eddies, J. Mar. Res., 33, 1–13, 1975.

Venegas, S. A.: The Antarctic Circumpolar Wave: A Combination of Two Signals?, J. Climate, 16, 2509–2525, 2003.

White, W. B. and Peterson, R. G.: An antarctic circumpolar wave in surface pressure, temperature and sea-ice extent, Nature, 380, 699–702, 1996.