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Nonlinear viscoelastic compaction in sedimentary basins

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Abstract. In the mathematical modelling of sediment compaction and porous media flow, the rheological behaviour of sediments is typically modelled in terms of a nonlinear relationship between effective pressure p_e and porosity ϕ, that is $p_e = p_e(\phi)$. The compaction law is essentially a poroelastic one. However, viscous compaction due to pressure solution becomes important at larger depths and causes this relationship to become more akin to a viscous rheology. A generalised viscoelastic compaction model of Maxwell type is formulated, and different styles of nonlinear behaviour are asymptotically analysed and compared in this paper.

1 Introduction

When well-bores are being drilled for oil exploration, drilling mud is used in the hole to maintain its integrity and safety. The mud density must be sufficient to prevent collapse of the hole, but not so high that hydrofracturing of the surrounding rock occurs. Both these effects depend on the pore fluid pressure in the rock, and drilling problems occur in regions where abnormal pore pressure or *overpressuring* occurs, that is in the regions, normally in the sedimentary basins such as the North Sea, where pore pressure increases downward faster than hydrostatic pressure. Such kind of overpressuring can substantially affect oil-drilling rates and even cause serious blowouts during drilling. Therefore, an industrially important objective is to predict overpressuring before drilling and to identify its precursors during drilling. Another related objective is to predict reservoir quality and hydrocarbon migration. An essential step to achieve such objectives is the scientific understanding of their mechanisms and the evolutionary history of post-depositional sediments such as shales.

Shales and other fine-grained compressible rocks are considered to be the source rocks for much petroleum found in sandstones and carbonates. At deposition, sediments such as shales and sands typically have porosities of order $0.5 \sim 0.75$ (Lerche, 1990). When sediments are drilled at a depth, say 5000 m, porosities are typically $0.05 \sim 0.2$. Thus an enormous amount of water has escaped from the sediments during their deposition and later evolution. Because of the fluid escape, the grain-to-grain contact pressure must increase to support the overlying sediment weight. Dynamical fluid escape depends lithologically on the permeability behavior of the evolving sediments. As fluid escape proceeds, porosity decreases, so permeability becomes smaller, leading to an ever-increasing delay in extracting the residual fluids. The addition of more overburden sediments is then compensated for by an increase of excess pressure in the retained fluids. Thus overpressure develops from such a *non-equilibrium compaction* environment (Audet and Fowler, 1992; Fowler and Yang, 1998). A rapidly accumulating basin is unable to expel pore fluids sufficiently rapidly due to the weight of overburden rock. The development of overpressuring retards compaction, resulting in a higher porosity, a higher permeability and a higher thermal conductivity than are normal for a given depth, which changes the structural and stratigraphic shaping of sedimentary units and provides a potential for hydrocarbon migration. Therefore, the determination of the mechanism of dynamical evolution of fluid escape and the timing of oil and gas migration out of such fine-grained rocks is a major problem. The fundamental understanding of mechanical and physico-chemical properties of these rocks in the earth's crust has important applications in petrology, sedimentology, soil mechanics, oil and gas engineering and other geophysical research areas.

Compaction is the process of volume reduction via pore-water expulsion within sediments due to the increasing weight of overburden load. The requirement of its occurrence is not only the application of an over-

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burden load but also the expulsion of pore water. The extent of compaction is strongly influenced by burial history and the lithology of sediments. The freshly deposited loosely packed sediments tend to evolve, like an open system, towards a closely packed grain framework during the initial stages of burial compaction and this is accomplished by the processes of grain slippage, rotation, bending and brittle fracturing. Such reorientation processes are collectively referred to as *mechanical compaction*, which generally takes place in the first 1 - 2 km of burial. After this initial porosity loss, further porosity reduction is accomplished by the process of *chemical compaction* such as pressure solution (Fowler and Yang, 1999).

Despite the importance of compaction and diagenesis for geological problems, the literature of quantitative modelling is not a huge one though the processes have received much attention in the literature, and the mechanism leading to pressure solution is still poorly understood. The effect of gravitational compaction was reviewed by Hedberg (1936) who suggested that an interdisciplinary study involving soil mechanics, geochemistry, geophysics and geology is needed for a full understanding of the gravitational compaction process. More comprehensive and recent reviews on the subject of compaction of argillaceous sediments were made by Rieke and Chilingarian (1974) and Fowler and Yang (1998).

Compaction is widely speaking a density-driven flow in a porous medium, which is a fascinating multidisciplinary topic that has attracted attention from scientists with different expertise for a long time. Holzbercher (1998) provides a very up-to-date comprehensive review of the previous works and state-of-art numerical methods and softwares for modeling density-driven flow and transport in porous media where the constant porosity is used. However, we will mainly model how porosity changes with time and depth rather than using a constant density, thus an appropriate compaction relation is vitally important.

Nonlinear compaction models have been formulated in two ways in terms of compaction relations. The early and most models used elastic or poroelastic rheology, and the compaction relation is Athy's type $p_e = p_e(\phi)$ (Gibson, England & Hussey, 1967; Smith, 1971; Sharp, 1976; Wangen, 1992; Audet and Fowler, 1992; Fowler and Yang, 1998). The more recent models begin to use viscous rheology with a compaction relation $p_e = -\xi\nabla.\mathbf{u}^s$ (Angevine and Turcotte, 1983; Birchwood and Turcotte, 1994; Fowler and Yang, 1999). The poroelastic models are valid for the mechanical compactions while the viscous models are mainly describing the chemical compaction such as pressure solutions. More recently, efforts have been made to give a more realistic visco-elastic model (Revil, 1999; Fowler and Yang, 1999). Fowler and Yang (1999) uses a viscous law $p_e = -\xi\nabla.\mathbf{u}^s$ to model compaction due to pressure solution, while Revil (1999) uses a poro-visco-plastic model, with a relationship between porosity strain and effective stress, to study pressure solution mechanism and its applications. However, there is no viscoelastic model which has been formulated to analyse the compaction problem on a basin scale, and most of the conventional studies are mainly numerical simulations. Obviously more work is yet to be done. This paper aims at providing a unified approach to the compaction relation by using a visco-poroelastic relation of Maxwell type. The nonlinear partial differential equations are then analysed by using asymptotic methods and the obtained analytical solutions are compared with numerical simulations.

2 MATHEMATICAL MODEL

For the convenience of investigating the effect of sediment compaction, we will assume a single species only. The sediments act as a compressible porous matrix, so that mass conservation of pore fluid together with Darcy's law leads to the model equations of the general type.
Conservation of mass

$$\frac{\partial}{\partial t}(1-\phi) + \nabla \cdot [(1-\phi)\mathbf{u}^s] = 0, \tag{1}$$

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{u}^l) = 0, \tag{2}$$

Darcy's law

$$\phi(\mathbf{u}^l - \mathbf{u}^s) = -\frac{k}{\mu}(\nabla p^l + \rho_l g \mathbf{j}), \tag{3}$$

Force balance

$$\nabla \cdot \boldsymbol{\sigma}^e - \nabla[p^l] - \rho g \mathbf{j} = \mathbf{0}, \tag{4}$$

where $\mathbf{u}^l$ and $\mathbf{u}^s$ are the velocities of fluid and solid matrix, k and μ are the matrix permeability and the liquid viscosity, ρ_l and ρ_s are the densities of fluid and solid matrix, $\boldsymbol{\sigma_e}$ is the effective stress, p_e is the effective pressure, $\mathbf{j}$ is the unit vector pointing vertically upwards, p^l is the pore pressure, and g is the gravitational acceleration. In addition, a rheological compaction law is needed to complete this model.

2.1 Poroelasticity and Viscous Compaction

The compaction law is a relationship between effective pressure p_e and strain rate $\dot{e}$ or porosity ϕ. The common approach in soil mechanics and sediment compaction is to model this generally nonlinear behaviour as poroelastic, that is to say, a relationship of Athy's law type $p_e = p_e(\phi)$, which is derived from fitting the real data of sediments. Athy's poroelasticity law is also a simplified form of Critical State Theory (Schofield and Wroth, 1986). A common relation representing the poroelasticity is

$$\frac{Dp_e}{Dt} = -K\nabla \cdot \mathbf{u}^s, \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}^s \cdot \nabla, \tag{5}$$

and equation (12) can be rewritten as

$$\frac{1}{1-\phi}\frac{D(1-\phi)}{Dt} = -\nabla \cdot \mathbf{u}^s, \tag{6}$$

combining with the previous equation, we have

$$p_e = p_e(\phi), \tag{7}$$

which is the Athy's law for poroelasticity. A typical form of this constitutive relation (Smith, 1971; Audet and Fowler, 1992; Fowler and Yang, 1998) is

$$p_e = \ln(\phi_0/\phi) - (\phi_0 - \phi). \tag{8}$$

However, this poroelastic compaction law is only valid for the sediment compaction in the upper and shallow region, where compaction occurs due to the pure mechanical movements such as grain sliding and packing rearrangement. In the deeper region, mechanical compaction is gradually replaced by the chemical compaction due to stress-enhanced flow along the grain boundary from the grain contact areas to the free pore, where pressure is essentially pore pressure. A typical process of such chemical compaction in sediment is pressure solution whose rheological behavior is usually viscous, so that it is sometimes called viscous pressure solution or viscous creep.

The mathematical formulation of compaction laws for pressure solution is to derive the creep rate in terms of concentrations, grain size and geometry (usually spherical or cylindrical packings), effective stress, grain boundary diffusion. This allows us to include the detailed reaction-transport process in a simplified relation between strain rate and effective stress although further simplifications are usually assumed such as steady-state dissolution and *local* reprecipitation along the grain boundary. Rutter's *creep law* (1976) is widely used

$$\dot{e} = \frac{A_k c_0 w D_{gb}}{\rho_s \bar{d}^3}\sigma, \tag{9}$$

where σ is the effective normal stress across the grain contacts, A_k is a constant, c_0 is the equilibrium concentration (of quartz) in pore fluid, ρ, $\bar{d}$ are the density and (averaged) grain diameter (of quartz). D_{gb} is the diffusivity of the solute in water along grain boundaries with a thickness w. Note that $p_e = -\sigma$ and $\dot{e}_{kk} = \nabla \cdot \mathbf{u^s}$. With this, (9) becomes the following compaction law

$$p_e = -\xi \nabla . \mathbf{u}^s. \tag{10}$$

This was first used by Birchwood and Turcotte (1994) to study pressure solution in sedimentary basins by presenting a unified approach to geopressuring, low permeability zone formation and secondary porosity generation.

2.2 1-D Viscoelastic Compaction

Following the discussions of elastic compaction (Fowler and Yang 1998) and viscous compaction (Fowler and Yang, 1999), we can generalise the above relations to a viscoelastic compaction law of Maxwell type

$$\boldsymbol{\nabla} . \mathbf{u}^s = -\frac{1}{K}\frac{Dp_e}{Dt} - \frac{1}{\xi}p_e. \tag{11}$$

Equivalently, we would anticipate a viscoelastic rheology for the medium, involving material derivatives of tensors, and some care is needed to ensure that the resulting model is frame indifferent. That is to say, the rheological relation of stress-strain should be invariant under the coordinate transformation. This is not always guaranteed due to the complexity of the rheological relations (Bird, Armstrong & Hassager 1977). Fortunately, for one-dimensional flow, which is always *irrotational*, the equation is invariant and all the different equations in corotational and codeformational frames degenerate into the same form (Yang, 1997). In the one-dimensional case we will discuss below, we can take this for granted. Thus the 1-D model equations become

$$\frac{\partial(1-\phi)}{\partial t} + \frac{\partial}{\partial z}[(1-\phi)u^s] = 0, \tag{12}$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial(\phi u^l)}{\partial z} = 0, \tag{13}$$

$$\phi(u^l - u^s) = \frac{\hat{k}(\phi)}{\mu}[-G\frac{\partial p_e}{\partial z} - (\rho_s - \rho_l)(1-\phi)g], \tag{14}$$

$$\frac{\partial u^s}{\partial z} = -\frac{1}{K}\frac{Dp_e}{Dt} - \frac{1}{\xi}p_e, \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + u^s\frac{\partial}{\partial z}, \tag{15}$$

where $G = 1 + 4\eta/3\xi$ is a constant describing sediment properties, and η is medium viscosity. By assuming the densities ρ_s and ρ_l are constants, we can see that only the density difference $\rho_s - \rho_l$ is important to the flow evolution. Thus, the compactional flow is essentially density-driven flow in a porous medium (Holzbercher, 1998).

3 Non-dimensionalization

If a length-scale d is a typical length defined by

$$d = \{\frac{\xi \dot{m}_s G}{(\rho_s - \rho_l)g}\}^{1/2}, \tag{16}$$

so that the dimensionless pressure $p = Gp_e/(\rho_s - \rho_l)gd = O(1)$. Meanwhile, we scale z with d, u^s with $\dot{m}_s$, time t with $d/\dot{m}_s$, permeability k with k_0. By writing $k(\phi) = k_0 k^*$, $z = dz^*$, ..., and dropping the asterisks, we thus have

$$-\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}[(1-\phi)u^s] = 0, \tag{17}$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial(\phi u^l)}{\partial z} = 0, \quad (18)$$

$$\phi(u^l - u^s) = \lambda k(\phi)[-\frac{\partial p}{\partial z} - (1-\phi)], \quad (19)$$

$$\frac{\partial u^s}{\partial z} = -\frac{\phi}{(1-\phi)^2}\frac{Dp}{Dt} - p, \quad \frac{D}{Dt} = \frac{\partial}{\partial t} + u^s\frac{\partial}{\partial z}, \quad (20)$$

where

$$\lambda = \frac{k_0(\rho_s - \rho_l)g}{\mu \dot{m}_s}. \quad (21)$$

In the above derivation, we have used the requirement of degenerating to the poroelastic case (8) when neglecting viscous rheology.

Adding (17) and (18) together and integrating from the bottom, we have

$$u^s = -\phi(u^l - u^s), \quad (22)$$

where $u = \phi(u^l - u^s)$ is the Darcy flow velocity. By using the equation (6), we now have

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z}[(1-\phi)u^s], \quad (23)$$

$$u^s = \lambda(\frac{\phi}{\phi_0})^m[-\frac{\partial p}{\partial z} - (1-\phi)]. \quad (24)$$

$$\frac{1}{(1-\phi)}\frac{D\phi}{Dt} = -\frac{\phi}{(1-\phi)^2}\frac{Dp}{Dt} - p. \quad (25)$$

The constitutive relation for permeability $k(\phi)$ is nonlinear, and its typical form is

$$k(\phi) = (\frac{\phi}{\phi_0})^m, \quad m = 8. \quad (26)$$

The boundary conditions at $z = 0$ are

$$\frac{\partial p}{\partial z} - (1-\phi) = 0 \ \ \text{(or equivalently, } u^s = 0), \quad (27)$$

$$\phi = \phi_0, \ \ p = 0, \quad (28)$$

$$\dot{h} = \dot{m}(t) + \lambda(\frac{\phi}{\phi_0})^m[\frac{\partial p}{\partial z} - (1-\phi)] \ \ \text{at} \ \ z = h(t), \quad (29)$$

which is a moving boundary problem.

It is useful to estimate these parameters by using values taken from observations. By using the typical values of $\rho_l \sim 10^3\,\mathrm{kg\,m^{-3}}$, $\rho_s \sim 2.5\times 10^3\,\mathrm{kg\,m^{-3}}$, $k_0 \sim 10^{-15} - -10^{-20}\,\mathrm{m^2}$, $\mu \sim 10^{-3}\,\mathrm{N\,s\,m^2}$, $\xi \sim 1\times 10^{21}$ N s m^{-2}, $\dot{m}_s \sim 300\,\mathrm{m\,Ma^{-1}} = 1\times 10^{-11}$ m s^{-1}, $g \sim 10\mathrm{m\,s^{-2}}$, $G \sim 1$, $d \sim 1000$m; then $\lambda \approx 0.01 \sim 1000$. We can see that the only parameter λ, which governs the evolution of the fluid flow and porosity in sedimentary basins, is the ratio between the permeability and the sedimentation rate.

4 Asymptotic Analysis

Since the nondimensional parameter $\lambda \approx 0.01 \sim 1000$ varies greatly and essentially controls the compaction process, we can expect that the two distinguished limits ($\lambda \ll 1$ and $\lambda \gg 1$) will have very different features in porosity and flow evolutions. In fact, $\lambda = 1$ defines a transition between slow compaction ($\lambda << 1$) and fast compaction ($\lambda >> 1$). Thus we can follow the similar asymptotic analysis (Fowler and Yang, 1998; Fowler and Yang, 1999) to obtain some analytical asymptotic solutions.

4.1 Slow Compaction ($\lambda \ll 1$)

In the case of $\lambda \ll 1$, $z \sim 1$, $t \sim 1$, $p \sim 1$ implies that $u^s \ll 1$ and $\frac{\partial \phi}{\partial t} \approx 0$, then $\phi \approx \phi_0$ and $D\phi/Dt \approx \frac{\partial \phi}{\partial t}$. We thus have

$$\frac{\partial \phi}{\partial t} \approx -\lambda(1-\phi_0)\frac{\partial^2 p}{\partial z^2}, \quad (30)$$

$$u^s \approx \lambda[-\frac{\partial p}{\partial z} - (1-\phi_0)], \quad (31)$$

$$\frac{1}{(1-\phi_0)}\frac{\partial \phi}{\partial t} \approx -\frac{\phi_0}{(1-\phi_0)^2}\frac{\partial p}{\partial t} - p, \quad (32)$$

which can be rewritten approximately as

$$\frac{\partial p}{\partial t} = \lambda'\frac{\partial^2 p}{\partial z^2} + \frac{(1-\phi_0)^2}{\phi_0}p, \quad \lambda' = \frac{(1-\phi_0)^2\lambda}{\phi_0}, \quad (33)$$

with appropriate boundary conditions

$$\frac{\partial p}{\partial z} \approx 1 - \phi_0, \quad \text{on} \quad z = 0, \quad (34)$$

$$p \to 0, \quad z \to \infty, \quad (35)$$

This is in fact equivalent to the case of conduction in a semi-infinite space with a constant flux at $z = 0$. The solution of this case can be approximately expressed as

$$p \approx (1-\phi_0)\sqrt{4\lambda' t}\ \mathrm{ierfc}(\zeta) + \sqrt{\lambda'\phi_0}\exp[-\frac{(1-\phi_0)z}{\sqrt{\lambda'\phi_0}}], \quad (36)$$

where

$$\zeta = \frac{z}{\sqrt{4\lambda' t}}, \quad (37)$$

and

$$\mathrm{ierfc}(\zeta) = \frac{1}{\sqrt{\pi}}e^{-\zeta^2} - \zeta\,\mathrm{erfc}(\zeta). \quad (38)$$

This gives an approximate solution of ϕ as

$$\phi \approx \phi_0 - \phi_0\sqrt{4\lambda' t}\ \mathrm{ierfc}(\zeta) - \frac{\phi_0\sqrt{\lambda'\phi_0}}{(1-\phi_0)}te^{-\frac{(1-\phi_0)z}{\sqrt{\lambda'\phi_0}}}, \quad (39)$$

We can see that compaction essentially occurs in a boundary layer near the bottom with a thickness of the order of $\sqrt{\lambda'}$. The comparison of this approximate solution (39)

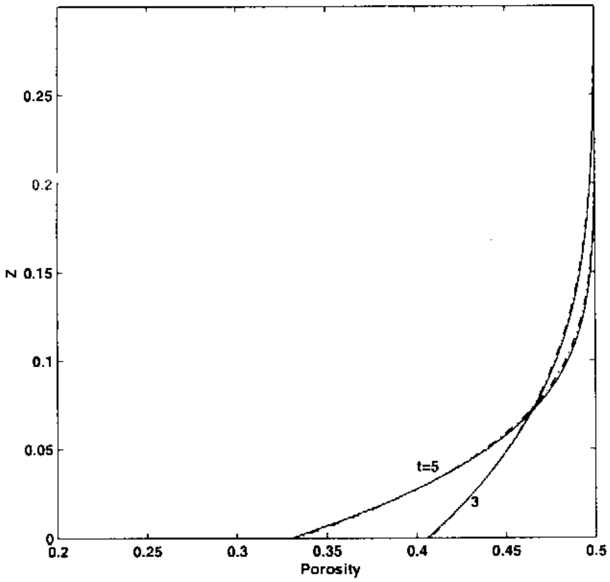

Fig. 1. Comparison of numerical solutions (solid curves) with asymptotic solution (39) for $\lambda = 0.01$ at $t = 3, 5$. Where $Z = z/h(t)$ is the scaled height.

(dashed curves) with numerical solutions (solid curves) is shown in Figure 1 for the values of $\lambda = 0.01$. $t = 5$. The approximate solution is accurate when $(\phi/\phi_0)^m \ll 1$ so that $t \sim 1/m\sqrt{\lambda} \sim 5$ due to the fact that $\phi(z = 0) = 1 - O(\sqrt{\lambda}t)$. The agreement is clearly shown in the figure.

4.2 Fast Compaction ($\lambda \gg 1$)

Fast compaction, either viscous or poroelastic, is more complicated and interesting in contrast to the simple structure of boundary layer for slow compaction. Since $\lambda \gg 1$ and the highly nonlinear permeability function $k = (\phi/\phi_0)^m$, $m = 8$, the governing equations are also highly nonlinear. However, we can use these features and pursue asymptotic analysis to seek appropriate asymptotic solutions.

4.3 Poroelastic Compaction

For the case $\lambda \gg 1$, we can rewrite (25) as

$$\frac{\partial \phi}{\partial t} + \lambda(\frac{\phi}{\phi_0})^m[-\frac{\partial p}{\partial z} - (1-\phi)]\frac{\partial \phi}{\partial z} = -(1-\phi)p$$

$$-\frac{\phi}{(1-\phi)}\{\frac{\partial p}{\partial t} + \lambda(\frac{\phi}{\phi_0})^m[-\frac{\partial p}{\partial z} - (1-\phi)]\frac{\partial p}{\partial z}\}, \qquad (40)$$

By using the perturbations

$$\phi = \phi^{(0)} + \frac{1}{\lambda}\phi^{(1)} + ..., \quad p = p^{(0)} + \frac{1}{\lambda}p^{(1)} + ..., \qquad (41)$$

the leading order equation becomes

$$\frac{\partial \phi^{(0)}}{\partial z} = -\frac{\phi^{(0)}}{(1-\phi^{(0)})}\frac{\partial p^{(0)}}{\partial z}, \qquad (42)$$

whose integration gives

$$p^{(0)} = \ln(\phi_0/\phi^{(0)}) - (\phi_0 - \phi^{(0)}), \qquad (43)$$

which is the Athy-type relation and is exactly the same form used by Smith (1971) and Fowler and Yang (1998). The leading order equation from (24) is thus

$$\frac{(1-\phi^{(0)})}{\phi^{(0)}}\frac{\partial \phi^{(0)}}{\partial z} - (1-\phi^{(0)}) = 0, \qquad (44)$$

or

$$\frac{\partial \phi^{(0)}}{\partial z} - \phi^{(0)} = 0. \qquad (45)$$

The appropriate boundary conditions $\phi^{(0)} = \phi_0$ gives

$$\phi^{(0)} = \phi_0 e^{-(h-z)}, \qquad (46)$$

which decreases with depth $h - z$ exponentially. This solution is the same as the equilibrium solution in the poroelastic case, and thus the top region of viscoelastic compaction is essentially poroelastic and viscous effect is only of secondary importance in this region. However, as ϕ decreases, the term $\lambda(\phi/\phi_0)^m$ may become very small due to the higher exponent $m = 8$. Naturally, $\lambda(\phi/\phi_0)^m = 1$ defines a critical value of ϕ in the transition region

$$\phi^* = \phi_0 e^{-\frac{\ln \lambda}{m}}. \qquad (47)$$

In fact, the above solutions are only valid when $\phi^{(0)} > \phi^*$ and $h - z < \Pi = (\ln \lambda)/m$.

4.4 Transition Region

As $\phi \sim \phi^*$, we define

$$\phi = \phi^* e^{\frac{\psi - \ln m}{m}}, \quad z = h - \Pi + \frac{\eta - \ln m}{m}, \qquad (48)$$

$$u^s = \frac{W}{m}, \quad p = p^* - \frac{P}{m}, \qquad (49)$$

where $p^* = \ln(\phi_0/\phi^*) - (\phi_0 - \phi^*)$. By changing variables to (t, η) via $\partial_t \to \partial_t - m\dot{h}\partial_\eta$, $\partial_z \to m\partial_\eta$, and assuming $m \gg 1$, we have the leading order equations

$$\dot{h}\phi^*\psi_\eta + (1-\phi^*)W_\eta = 0, \qquad (50)$$

$$W = e^{\psi}[P_\eta - (1-\phi^*)], \qquad (51)$$

$$\dot{h}\phi^*\psi_\eta = \frac{\phi^*\dot{h}}{(1-\phi^*)}P_\eta + (1-\phi^*)p^*. \qquad (52)$$

Thus

$$W = W^* - \frac{\dot{h}\phi^*}{(1-\phi^*)}\psi, \qquad (53)$$

$$\psi_\eta = 1 - \frac{(1-\phi^*)p^*}{\dot{h}\phi^*} + (\psi_\infty - \psi)e^{-\psi}. \qquad (54)$$

$$\psi_\infty = \frac{(1-\phi^*)W^*}{\dot{h}\phi^*}, \tag{55}$$

whose solution can be written as a quadrature. As $\eta \to -\infty$, $P_\eta \to 0$, we can see in equation (52) that the dominant term is the viscous term $(1-\phi^*)p^*$ so that compaction will gradually transfer from viscoelastic one to purely viscous one. This has important geophysical implication for compaction in sedimentary basins, since the purely viscous mechanism may be responsible for overpressuring and mineralized seals in oil-reservoir and hydrocarbon basins. Furthermore, In order to determine $\dot{h}$, we require (53) and (55) to match the solution below the transition layer.

4.5 Viscous Compaction

In the region below the transition layer, porosity $\phi < \phi^*$ is usually very small, while the effective pressure p is increasing and $p \sim p^* = O(1)$. Rewriting (25) as

$$\frac{\partial u^s}{\partial z} = -\frac{\phi}{(1-\phi)^2}[\frac{\partial p}{\partial t} + u^s\frac{\partial p}{\partial z}] - p, \tag{56}$$

From (49) and (52), we know that p changes slowly, which implies $\frac{\partial p}{\partial t} \sim \frac{\partial p}{\partial z} \ll 1$ or $\phi(\frac{\partial p}{\partial t} + u^s\frac{\partial p}{\partial z}) \ll p$, we then have approximately

$$p \approx -\frac{\partial u^s}{\partial z}, \tag{57}$$

which implies that compaction is now essentially purely viscous. Thus we get

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z}[(1-\phi)u^s], \tag{58}$$

$$u^s = \lambda(\frac{\phi}{\phi_0})^m[\frac{\partial^2 u^s}{\partial z^2} - (1-\phi)], \tag{59}$$

which are the equations solved by Fowler and Yang (1999) when $\Xi = 1$ for purely viscous compaction. Following the same solution procedure given by Fowler and Yang (1999), we can expect to get the same solutions. Thus, we only write down here the solution for $\dot{h}$

$$\dot{h} = \dot{m}_s[\frac{(1-\phi_0)}{(1-\phi^*)} + \frac{\phi^*\psi_\infty}{m(1-\phi^*)}] - \frac{2p^*}{\gamma m}\ln m, \tag{60}$$

where $\gamma = \frac{p^*(1-\phi^*)^2}{\dot{m}_s\phi^*(1-\phi_0)}$. This essentially completes the solution procedure. Figure 2 shows the comparison of numerical results with the above obtained asymptotic solutions (46) and (55) in the poroelastic and transition region.

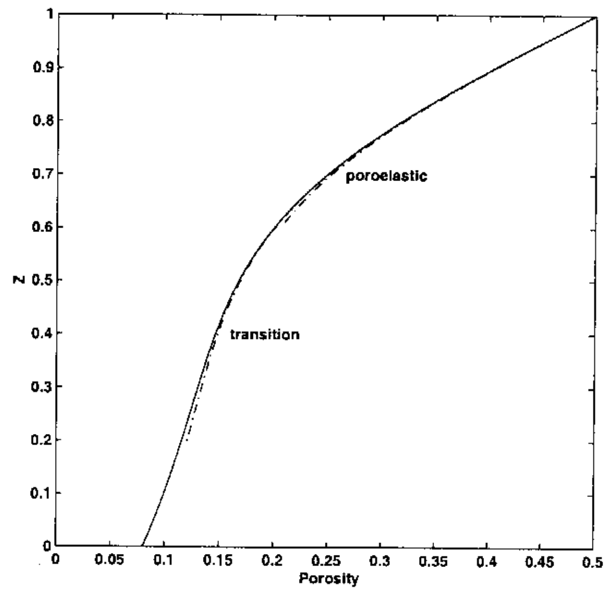

Fig. 2. Numerical results (solid curves) with $\lambda = 100$, $t = 10$. The asymptotic solutions (46) and (55) are also plotted as a comparison (dashed). Profile in the top region is nearly exponential follows by a transition to pure viscous compaction where porosity is nearly uniform.

5 Discussion

The present model of viscoelastic flow and nonlinear compaction in sedimentary basins uses a rheological relation which incorporates both poroelastic and viscous effects in 1-D compacting frame. Based on the frame invariance of irrotational feature of the 1-D flow, a generalised viscoelastic compaction relation of Maxwell type has been formulated. The nondimensional model equations are mainly controlled by one parameter λ, which is the ratio of hydraulic conductivity to the sedimentation rate. Following the similar asymptotic analysis given by Fowler and Yang (1998), we have been able to obtain the approximate solutions for either slow compaction ($\lambda \ll 1$) or fast compaction ($\lambda \gg 1$). The more realistic and yet more interesting case is when $\lambda \gg 1$, and the solution implies a nearly exponential profile of porosity versus depth, which implies that compaction in the top region is essentially poroelastic and its profile is virtually at equilibrium.

The numerical simulations and asymptotic analysis have shown that porosity-depth profile is near exponential followed by a a transition from poroelastic to viscoelastic region. This is because of the large exponent m in the permeability law $k = (\phi/\phi_0)^m$, so that even if $\lambda \gg 1$, the product λk may become small at sufficiently large depths. In this case, the porosity profile consists of an upper part near the surface where $\lambda k \gg 1$ and the equilibrium is attained, and a lower part where $\lambda k \ll 1$, and the porosity is higher than equilibrium which appears to correspond accurately to numerical computations. Below this transition region, porosity is usually uniformly small and compaction is essentially pure viscous. From the definition of excess pore pressure $p_{ex} = \int_h^z [p + (1-\phi)]dz$, we know that the sudden switch from poroelastic to viscous compaction means a quick decease of porosity ϕ, which leads to a sudden increase of p_{ex}. Therefore, the transition is often associated with a jump to a high pore pressure and low permeability region where a mineralized seal may be formed. This conclusion is consistent with the earlier work (Birchwood and Turcotte, 1994). As viscous compaction proceeds, porosity and permeability may become so small that fluid gets trapped below this region, and compaction virtually stops.

Further work shall focus on more realistic and correct formulation of rheology. In a recent work on pressure solution and its application to some field problems such as land subsidence associated with fluid withdrawal from undercompacted aquifers, Revil (1999) suggests a Voigt-type poro-visco-plastic rheological behavior to characterize pressure solution and to applications to some field problems including equilibrium and disequilibrium compactions and subsidence. Naturally, more work is needed to incorporate a Voigt-type rheology applied to compaction in addition to a present Maxwell-type law.

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