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Anomalous diffusion in geophysical and laboratory turbulence

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Abstract. We present an overview and some new results on anomalous diffusion of passive scalar in turbulent flows (including those used by Richardson in his famous paper in 1926). The obtained results are based on the analysis of the properties of invariant quantities (energy, enstrophy, dissipation, enstrophy generation, helicity density, etc.) - i.e. independent of the choice of the system of reference as the most appropriate to describe physical processes - in three different turbulent laboratory flows (grid-flow, jet and boundary layer, see Tsinober et al. (1992) and Kit et al. (1993)).
The emphasis is made on the relations between the asymptotic properties of the intermittency exponents of higher order moments of different turbulent fields (energy, dissipation, helicity, spontaneous breaking of isotropy and reflexional symmetry) and the variability of turbulent diffusion in the atmospheric boundary layer, in the troposphere and in the stratosphere. It is argued that *local spontaneous breaking of isotropy of turbulent flow results in anomalous scaling laws for turbulent diffusion* (as compared to the scaling law of Richardson) *which are observed, as a rule, in different atmospheric layers from the atmospheric boundary layer (ABL) to the stratosphere. Breaking of rotational symmetry is important in the ABL, whereas reflexional symmetry breaking is dominating in the troposphere locally and in the stratosphere globally.*
The results are of speculative nature and further analysis is necessary to validate or disprove the claims made, since *the correspondence with the experimental results may occur for the wrong reasons* as happens from time to time in the field of turbulence.

1 Introduction

Geophysical turbulent flows are characterized by rather large Reynolds numbers. Therefore, it has been a common expectation that universal relations (such as energy spectrum $E(k) \sim k^{-5/3}$, diffusivity $\mathcal{K} \sim \ell^{\,4/3}$) should be valid in such flows as well as their "two-dimensional" analogs in quasi-two-dimensional situations.
Richardson (1926) in his famous paper initiated the modern approach to the subject of turbulent diffusion (Taylor , 1959), stressing the importance of relative diffusion rather than single-particle diffusion. In particular, to find out how the coefficient of eddy diffusifity $\mathcal{K}$ varies with scale ℓ Richardson plotted $\mathcal{K}$ versus ℓ ranging from 0.05 to $10^8 cm$. His original plot is reproduced in Fig. 1a.
Discussing this result Taylor writes:
It will be seen that if the lowest point, which refers to molecular diffusion, and the highest point which refers to transfer over distances of thousands of kilometers are left out of consideration the staright line

$$\mathcal{K} = 0.2\ell^{\,4/3} \qquad (2)$$

is a very good approximation to the curve between $\ell = 10^2$ *and* $\ell = 10^6 cm$. *Since the curve shown here seems to contain all the observational data that Richardson had when he announced the remarkable law (2), it reveals a well-developed physical intuition that he chose as his index 4/3 instead of, say, 1.3 or 1.4 but he had the idea that the index was determined by something connected with the way energy was handed down from larger to smaller and smaller eddies. He perceived that this is a process which, because of its universality, must be subject to some simple universal rule. It is perhaps rather surprising that he did not take the step which Kolmogoroff (1941) and Obukhov took fifteen years later, namely to express his equation non-dimensionally using only the two physical quantities which could be relevant to a universal rule regulating the handling down of energy, namely* ϵ *the rate of energy dissipation and* ν *the dynamical viscosity.*

The Richardson law was *claimed* to be confirmed in a large number of experiments (Monin and Yaglom (1971,

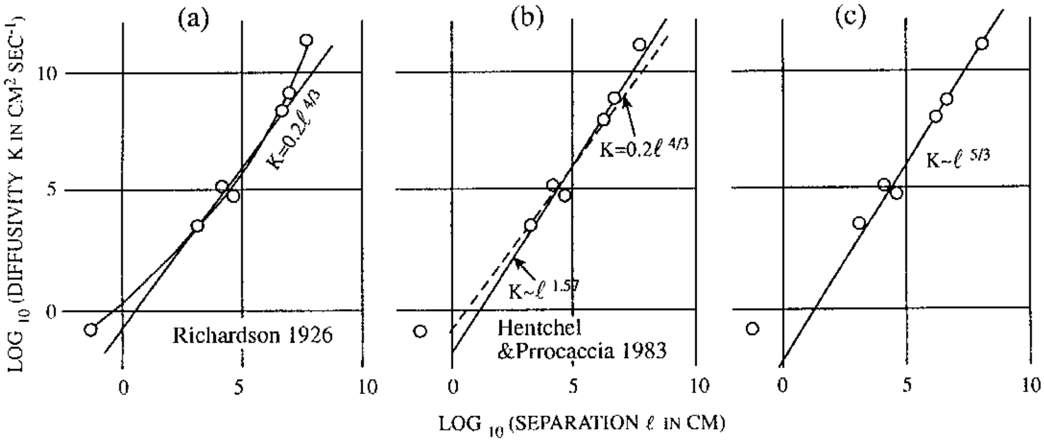

Fig. 1. Three interepretations of the data used by Richardson (1926).

1975), Monin and Ozmidov (1985)).

However, in spite of the common expectation there exist many examples of turbulent flows in the atmosphere, ocean and laboratory, in which the turbulent difussivity $\mathcal{K}$ as a function of scale ℓ **does not follow** the Richardson law (Richardson , 1926)

$$\mathcal{K} \sim \ell^{\,4/3}. \quad (1)$$

as well as its "two-dimensional" analog.[1] Examples of such behaviour are given in the main text of the paper for various situations. Here we give an example of different interpretation of the data of Richardson's original paper. Namely, it is claimed in Hentschel and Procaccia (1983) that, excluding the lowest point which pertains to molecular diffusivity, these data are best fitted by a relation

$$\mathcal{K} \sim \ell^{\,4/3+2\mu/3}, \quad (2)$$

with non-zero intermittency exponent $\mu = 0.36$ and a slope of 1.57 in 2 (see Fig. 1b adapted from Hentschel and Procaccia (1983)). However, this interpretation (as well as the original one by Richardson) does not take into account that the upper three points in Fig. 1 correspond to strongly anisotropic (quasi- two-dimensional – QTD) conditions. It is argued below that in such a situation the relevant parameter is the rate of production of helicity $\zeta = \langle | \, dh/dt \, | \rangle$ rather than ϵ the rate of energy dissipation. This results in the relation

$$\mathcal{K} \sim \ell^{\,5/3}. \quad (3)$$

with the exponent 5/3. The straight line with this slope is shown in Fig. 1c together with the data of Richardson's original paper.

[1] We refer to such situations as possessing anomalous diffusion (e.g., Zaslavsky , 1992). Recall the dimensionality of $[\mathcal{K}] = L^2T^{-1}$. For an overview and a partial list of references on 'misbehavoiur' of a passive scalar in turbulent flows see Holzer and Siggia (1994).

It is possible that more appropriate is a related quantity $\hat{\zeta} = \langle | \, d(\hat{h})/dt \, | \rangle$, where $\hat{h}_u = \hat{\mathbf{u}} \cdot \boldsymbol{\omega}$ and $\hat{\mathbf{u}} = \mathbf{u} + \nabla\phi$. It was shown by Kuzmin (1983) (see also Oseledets (1988)) that $\nabla\phi$ can be chosen in such a way that – in contrast to h – $\hat{h}$ is a lagrangian invariant, i.e. it is conserved along the paths (pointwise) and therefore for *any* fluid volume. In the absense of boundaries (or with some special boundary conditions) the integrals of h and $\hat{h}$ coinside. As long as one is concerned with dimensional arguments the result is the same employing either $\langle | \, dh/dt \, | \rangle$ or $\langle | \, d(\hat{h})/dt \, | \rangle$, since the dimensionality of both is the same.

One of the natural candidates among the possible reasons for the deviations from the Richardson law is the phenomenon of spontaneous breaking of statistical isotropy (rotational and/or reflexional) symmetry - locally or globally.[2] [3] In the sequel an attempt is made to provide a quantitative explanation of anomalous diffusion in terms of this phenomenon.

2 Atmospheric boundary layer

It is argued in Bershadskii et al. (1994) that regions with large fluctuations of turbulent energy are characterized by strong anisotropy and a local cascade of angular momentum (breaking of rotational symmetry), i.e. of a quantity of the type of Loytsianskii's invariant

[2] This should be distinguished from *imposed* reflexional symmetry breaking as in Cattaneo et al. (1988), Chechkin et al. (1993), Drummond et al. (1984), Moffatt (1983) and references therein.

[3] The intermittent (multi)fractal behaviour of turbulence could be considered as another possible reason (Hentschel and Procaccia , 1983). However, while both reasons seem to be intimately related (Tur and Levich , 1992) for the velocity field, there are some indications that the impact of (multi)fractality and intermittency on dispersion may be small (Borgas (1993), Borgas and Sawford (1993)).

$$\Lambda = \int_{\Omega_r} \langle \mathbf{u}(\mathbf{x})\mathbf{u}(\mathbf{x}+\mathbf{r})\rangle r^2 dr^{D_\infty}, \quad (4)$$

where D_∞ characterizes the subregions Ω_r with large fluctuations of turbulent energy

$$large \int_{\Omega_r} \mathbf{u}^2(\mathbf{x})d\mathbf{x} \sim r^{D_\infty}, \quad (5)$$

It is argued further in Bershadskii et al. (1994) that the governing parameter is the rate of transfer of angular momentum

$$\mathcal{L} = \mid \frac{d(\Lambda/V)}{dt} \mid, \quad (6)$$

which has the following dimensionality

$$[\mathcal{L}] = [L]^{1+D_\infty}[T]^{-3}. \quad (7)$$

It is straighforward to obtain the numerical value of D_∞ from dimensional arguments

$$large \int_{\Omega_r} \mathbf{u}^2(\mathbf{x})d\mathbf{x} \sim \mathcal{L}^{2/3} \; r^{13/5}, \quad (8)$$

i.e. that for the field of turbulent energy $D_\infty = 13/5$. These arguments are supported by laboratory and numerical data on asymptotic values of intermittency exponent μ_q for large q of turbulent energy (Bershadskii et al. (1994), Meneveau (1991), Hosokawa (1993)), which gave a value of $D_\infty = 2.6 \pm 0.05$.

In particular the parameter $\mathcal{L}$ becomes relevant in case when the energy of turbulence is supplied at different scales. In such a situation one can expect that the Richardson-Kolmogorov cascade process will not be realized since there will be not enough time to allow for the process of isotropization owing to the action of long range forces due to pressure gradients. However, the remaining anisotropy in such a case allows to assume that a 'cascade' of angular momentum mentioned above can be realized in a considerable range of scales. For example, one can expect such a 'cascade' in a turbulent flow over urban or rocky landscapes as well as over complex terrains.

A scaling relation for the effective diffusivity $\mathcal{K}$ as a function of scale ℓ follows from dimensional arguments assuming that the only relevant parameter is $\mathcal{L}$ (equation 8)

$$\mathcal{K} \sim \mathcal{L}^{1/3} \, \ell^{4/5}, \quad (9)$$

which is different from the Richardson law 1 as well from the relations describing correspondingly the initial ($\mathcal{K} \sim \ell$) and the final ($\mathcal{K} \sim$ *const*) stages of diffusion (Monin and Yaglom (1971, 1975), Pasquill and Smith (1983)). An estimate of the spread σ of a puff from a source of a passive scalar as a function of characteristic time of its motion τ can be found in a similar way[4]

[4] The exponents λ_1 and λ_2 in the relations $\mathcal{K} \sim \ell^{\lambda_1}$ and $\ell \sim t^{\lambda_2}$ are related by a simple relation $\lambda_1 = 2(\lambda_2 - 1)/\lambda_2$ or $\lambda_2 = 2/(2 - \lambda_1)$.

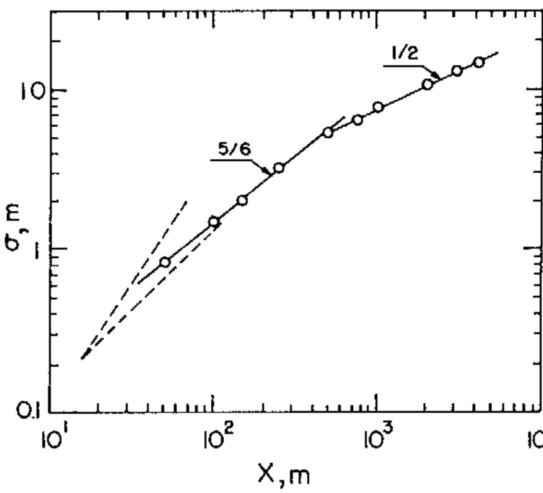

Fig. 2. Vertical spread from a source at a height 50 m at Agesta, Sweden measured by Högström 1964. Adapted from Pasquill (1983). The slope 5/6 corresponds to the relation $\mathcal{K} \sim \mathcal{L}^{1/3} \ell^{4/5}$.

$$\sigma \sim \mathcal{L}^{5/18} \, \tau^{5/6}. \quad (10)$$

In case when the puff is advected horizontally σ is taken from the vertical spread, while τ is estimated as X/V, where X is the distance from the source and V is the mean horizontal velocity (Monin and Yaglom (1971, 1975), p.365.) In such a case

$$\sigma \sim X^{5/6} \quad (11)$$

In Fig. 2 adapted from Pasquill and Smith (1983), p.218 are shown results obtained by Högström from a tube at a height of 50 m at Agesta, Sweden.

A straight line with the slope "5/6" is drawn in this figure in order to make a comparision with the relation 11 and also a straight line with the slope "1/2" corresponding to the long time limit in the statistical theory (Monin and Yaglom , 1971, 1975). The broken lines have the slopes "3/2" (Richardson-Kolmogorov theory) and "1" (short time limit in statistical theory). It is seen from the Fig. 2 that at the *initial stage* of the evolution of the puff of passive scalar it follows the relation 11, i.e. the process of turbulent diffusion seems to be controlled by the 'cascade' of angular momentum. A similar trend is seen clearly for a number of experimental results shown in Fig. 3 also adapted from Pasquill and Smith (1983), p.195.

While the above considerations can be applied to the initial stage of diffusion, in case of the final stage one has to take into account the presence of organized structures, which can modify considerably the process of turbulent diffusion (Bershadskii and Tsinober , 1993). It has been shown in Bershadskii et al. (1993b) that if in a turbulent flow there exist a finite number of large scale 'sinks' of turbulent energy (such as solitons, spotaneously formed large scale vortices, etc.) then at scales of the order of these objects it is more appropriate to

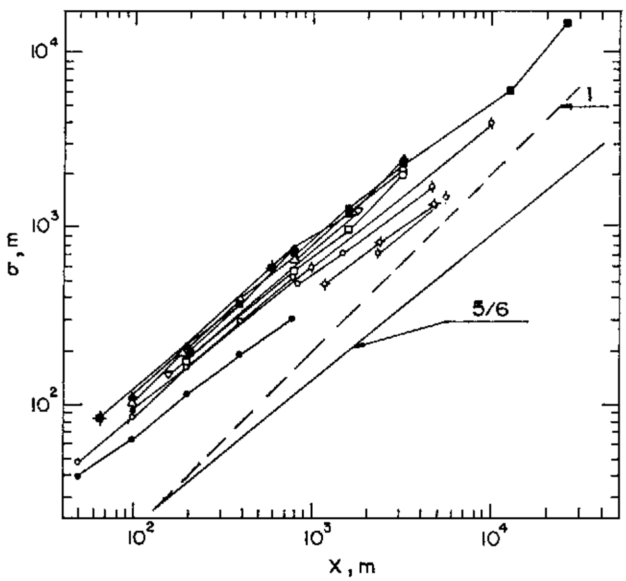

Fig. 3. Crosswind spread measured by different authors. Adapted from Pasquill (1983). The slope 5/6 corresponds to the relation $\mathcal{K} \sim \mathcal{L}^{1/3}\ \ell^{4/5}$.

use as a *governing parameter the 'dissipation' rate of energy per sink* - G and not the dissipation rate per volume unit $\langle\epsilon\rangle$ as in the Kolmogorov-Obukhov theory (Monin and Yaglom , 1971, 1975). Since G and $\langle\epsilon\rangle$ are of different dimensionality ($[G] = [L]^5[T]^{-3}$, whereas $[\langle\epsilon\rangle] = [L]^2[T]^{-3}$), it follows from dimensional arguments that the scaling relation for diffusivity has the following form

$$\mathcal{K} \sim G^{1/3}\ell^{\ 1/3} \tag{12}$$

Similarly

$$\sigma \sim\ G^{\ 1/5}\ t^{\ 3/5} \tag{13}$$

at the G-range of scales. In Fig. 4 (adapted form Pasquill and Smith (1983), p.225) is shown the vertical spread of elongated smoke puffs observed by Högström (1964) in Studswick (Sweden), source height 87m for two values of stability category $\Lambda = 2.25$ (lower points) and $\Lambda = 1.5$ (upper points).

We have drawn continuous straight lines with the slope 0.6 in this figure for compatison with the relation $\sigma\ \sim\ X^{\ 3/5}$ (where X is the horisontal distance from the source). The dotted lines correspond to the long time limit of statistical theory $\sigma\ \sim\ X^{\ 1/2}$ (see also Pasquill and Smith (1983), p.194 and Cramer et al. (1958)).

3 Diffusion in the troposphere and in the ocean

This question has been addressed in Bershadskii et al. (1993a) by means of analysis of experimental data on helicity obtained in laboratory for turbulent grid, boundary layer and jet flows (Tsinober et al. (1992), Kit et al.

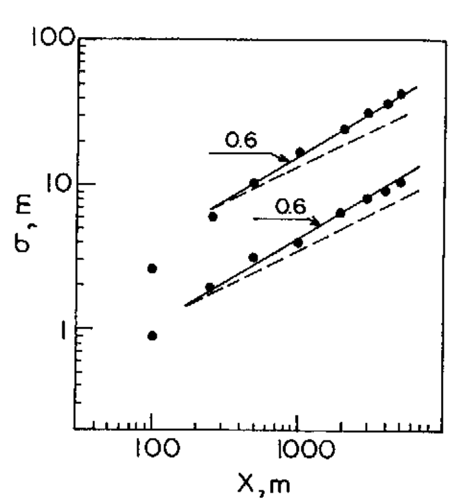

Fig. 4. Vertical spread of elongated smoke puffs in ABL. Adapted from Pasquill (1983).

(1993)). It was shown that Kolmogorov (homogeneous) turbulence is unstable in respect to local states - fractons (for their definition see Alexander (1986)), which appear to be the *subregions with large helicity*. These self-organized states arise spontaneously in subregions of turbulent flow with essential *breaking of reflexional symmetry* with large helicity. The governing dimensional parameter for helical fractons is different from the Kolmogorov one. It is the so called renormalized dissipation rate $\tilde{\epsilon}$ (Bershadskii et al. , 1993a), which has the dimensionality

$$\tilde{\epsilon} = [L^2][T]^{-1-D_f}, \tag{14}$$

with fracton dimension $D_f = 4/3$ (for details see Bershadskii et al. (1993a)).

In particular, the diffusivity $\mathcal{K}$ in fractons follows the relation

$$\mathcal{K} \sim \tilde{\epsilon}^{\ 3/7}\ell^{\ 8/7} \tag{15}$$

and not the law of Richardson 1.

In case, when the number of helical fractons is large enough , the mean diffusivity (over the whole flow region) will follow the relation 15 too. Such a possibility is rooted in the properties of fractons enabling them to trap the passive scalar inside them for a very long time. Therefore, after some initial period most of the passive scalar will be located within the fractons. On the other hand, the interaction of fractons with their environment is controlled primarily also by the parameter $\tilde{\epsilon}$, i.e this parameter controls the statistical properties of the stochastic trajectories of fractons. In other words, the statistical properties of the fractons trajectories will be determined mainly by the properties of the fractons themselves and to a much lesser degree by the properties of their environment. This brings us to the conclusion that the relation 15 can be valid not only on the scales of the order of fractons scales , but also in a range of

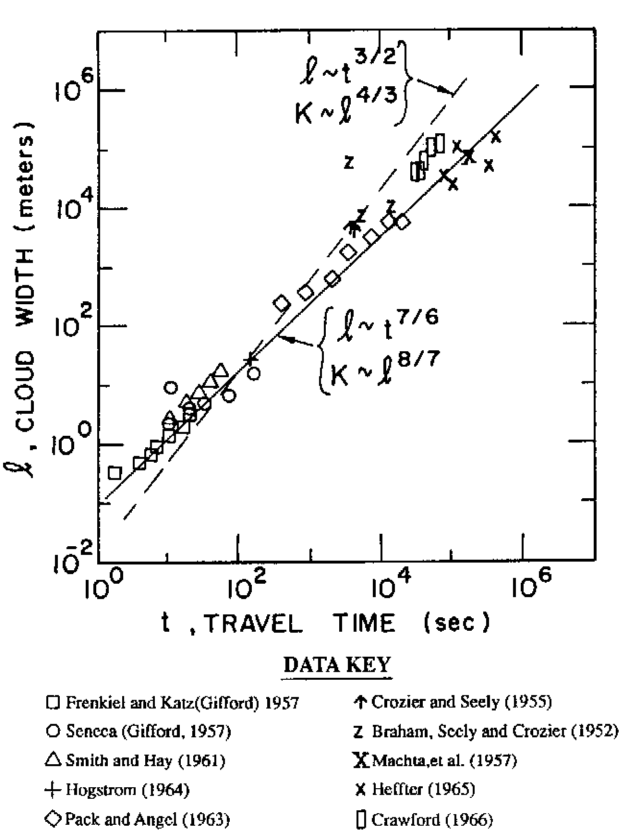

Fig. 5. Observations of widths (horizontal standard deviation) of diffusing tracer as a function of downwind travel time in troposphere. Different symbols correspond to the results of different authors. Adapted from Gifford (1983). The slope 7/6 corresponds to the relation $\mathcal{K} \sim \ell^{8/7}$.

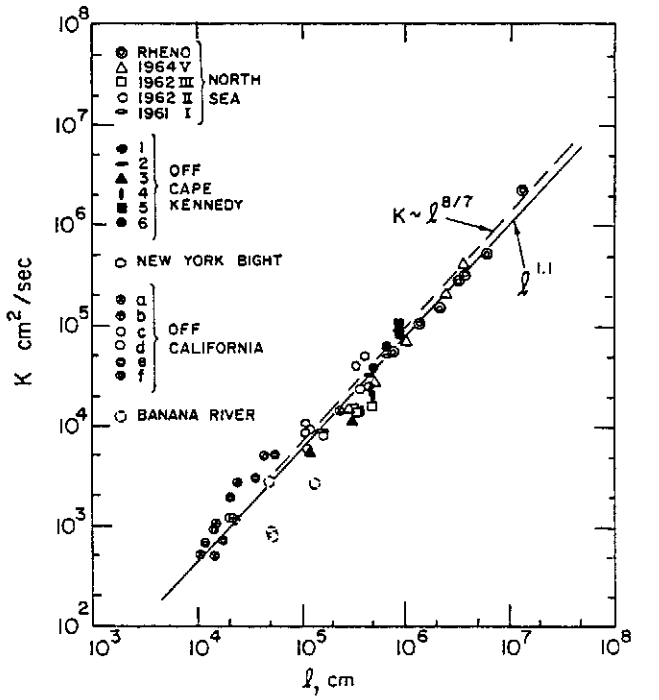

Fig. 6. Eddy diffusivity versus scale ℓ in the ocean. Adapted from Okubo (1971).

much larger scales. It is naturally to call this range the fracton range of scales. It is plausible that these properties of fractons form the basis of the extremely broad range of universal behaviour of the dependence $\ell(t)$ in the troposphere (see Fig. 5). Indeed, it follows from the relation 15 that

$$\ell \sim t^{7/6}. \qquad (16)$$

Looking at Fig. 5 - adapted from Gifford (1983) and containing data on cloud width versus travel time of many authors in different conditions - one is amazed that *all* these results are well described by a *single universal* relation 16 in the range of scales (horizontal standard deviation) from *one meter* to *one hundred kilometers.*

The straight line corresponds to the relation 15 and the dotted lines correspond to the relation 1 and $\mathcal{K} \sim \ell$. As seen from the Fig. 5 the universality of the relation 16 is manifested not only by the single exponent '7/6' but also by the *universal constant* in this relation. Apparently in all these experiments the fractons have been of the same type and the scales of cloud width fell into the fracton range.

The relation of $\mathcal{K}$ versus ℓ shown in Fig. 6 is based on the results obtained in the ocean by Okubo (1971) (see Fig. 7), where an empirical relation $\ell^2 \sim t^{2.34}$ was obtained.

The relation 15 results in $\ell^2 \sim t^{7/3}$.

The relations (15, 16) are valid also in *some cases* for quasi-two-dimensional turbulence (large horizontal scales in the troposphere - Fig. 5, and in the ocean - Fig. 6 and Fig. 7), since fractons - which are *three-dimensional* formations of rather small scale - most probably can be effective in the transport of a passive scalar on much larger quasi-two-dimensional scales for the same reasons as argued above.

Since the above results have been obtained in essentially different external conditions it is naturally to assume that the processes responsible for such universal behaviour are realized on spatially *localized (and compact) carrier with universal dynamics* (we call this process - *fracton transfer of a passive scalar*).

It should be stressed that the above results are rather speculative since in the atmosphere and in the ocean there are observed relations of $\mathcal{K}(\ell)$ different from 15 and the geophysical conditions leading to the fracton transfer of a passive scalar are not clear yet. In particular, in the next section an example of a different behaviour of turbulent diffusion in quasi-two-dimensional turbulence is given.

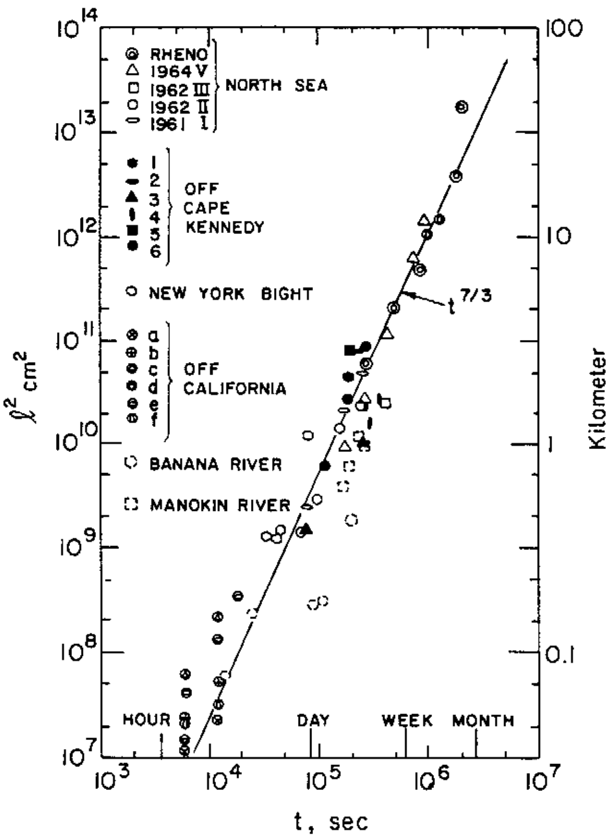

Fig. 7. Mean square separation ℓ versus time in the ocean. Adapted from Okubo (1971).

4 Diffusion in real quasi-two-dimensional turbulence - stratosphere

There exists a *qualitative* difference between strictly two-dimensional (2D) and real quasi-two-dimensional (Q2D) turbulence in spite of the "smallness" of the difference in their geometry. In fact, *this difference can be rather large primarily due to its topological nature.* In particular, helicity $\mathcal{H}_u = \int_V h_u dv$, helicity density $h_u = \mathbf{u} \cdot \boldsymbol{\omega}$, superhelicity $\mathcal{H}_\omega = \int_V h_\omega dv$ and its density $h_\omega = \boldsymbol{\omega} \cdot rot\ \boldsymbol{\omega}$ and related quantities [5] *vanish identically in strictly two-dimensional tubulence*, whereas in *real Q2D turbulence* $\mathcal{H}$ and h *can be finite for whatever small rate of change of flow properties along the slow variation coordinate.*

Since purely two-dimensional turbulence is unstable to three-dimensional perturbations it cannot be realized in real 3-D space. However, the 3-D instabilities can be moderated or even totally suppressed by external factors and constraints such as stratification, rotation, magnetic field, rigid walls or strong velocity gradient in some direction. It is argued in Bershadskii et al. (1993b) that in the presence of such factors the quasi-two-dimensional regime arises as a result of a spontaneous breaking of reflexional symmetry (parity breaking bifurcation), which in turn is a consequence of the instability of two-dimensional turbulence to three-dimensional helical travelling waves and/or solitons through super- and/or sub-critical bifurcations. Such instabilities can be realized on scales r_1 *much larger* than the the characteristic scale r_0 of energy input into the two-dimensional turbulent flow.[6]

The only source of energy for the 3-D disturbances is the basic two- dimensional turbulent flow with an energy input at the scale r_0. Since the characteristic scale of the travelling waves $r_1 >> r_0$ there should occur an inverse (anisotropic) energy transfer to support their existence. This energy transfer cannot be of a cascade type, due to the scale separation $r_1 >> r_0$. For this reason the mean rate of energy transfer $\langle\epsilon\rangle$ is not a governing parameter in this range of scales and its place it taken by the mean magnitude of rate of spontaneous helicity generation

$$\zeta = \langle | \, dh/dt \, | \rangle. \tag{17}$$

Then in the range $r_1 >> r >> r_0$ in analogy with the Kolmogorov theory it follows from dimensional arguments that the energy spectrum has the following form:

$$E_u(k) \sim \zeta^{2/3}\, k^{-7/3}, \tag{18}$$

where k is the modulus of the wave number in the plane of the primary two dimensional turbulent flow.

The expression 18 was obtained by Brissaud et al. (1973) for the case of *three*-dimensional isotropic turbulence. However, since in the last case there seems to exist no natural mechanism of scale separation r_0 and r_1 (see above) the expression 18 appeared to be inadequate to the existing experimental data. By contrast in the case of quasi-two-dimensional turbulence there is a great variety of experimental and field observations of spectra with wide ranges in full agreement with 18. We will limit ourselves with examples in which the difference between 2D and Q2D turbulence is manifested in particular in dissimilar difusive properties. Arguments similar to those used by Corrsin and Obukhov (Monin and Yaglom (1971, 1975) , p.377) lead to a following expression for the spectrum of fluctuations of a passive scalar

$$E_c(k) \sim \langle N\rangle \zeta^{-1/3}\, k^{-4/3}, \tag{19}$$

where $N =| \, dc^2/dt \, |$ An example of spectra of kinetic energy and temperature from the GASP flights in the stratosphere is shown in Fig. 8 (Gage and Nastrom , 1986a).

[5]For a review on helicity in laminar and tubulent flows see Moffatt and Tsinober (1992).

[6]It is noteworthy that the situation is different in the case of 3-D instability of laminar flows. Here, short wave instability can play an essential role due to the absence of (2-D) turbulent diffusion and of a stabilizing factor (Pierrehumbert (1986), Waleffe (1990)).

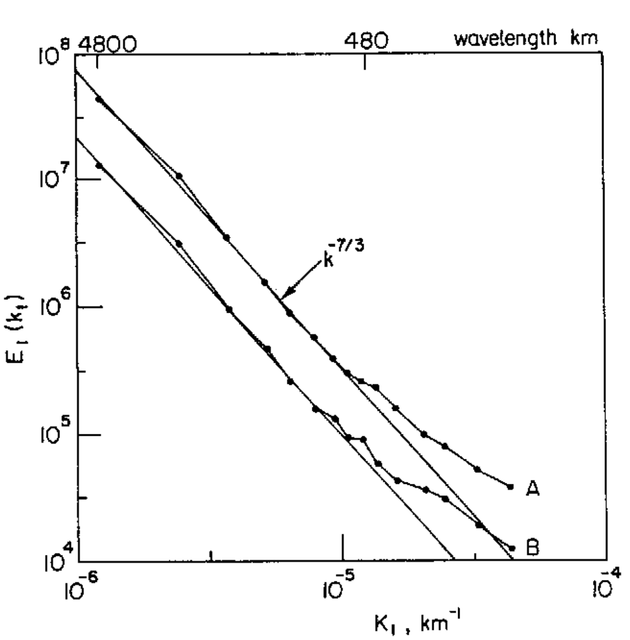

Fig. 8. Spectra from the GASP flights in stratosphere at least 4800 km long: A-kinetic energy; B-temperature. Adapted from Gage and Nastrom (1986a).

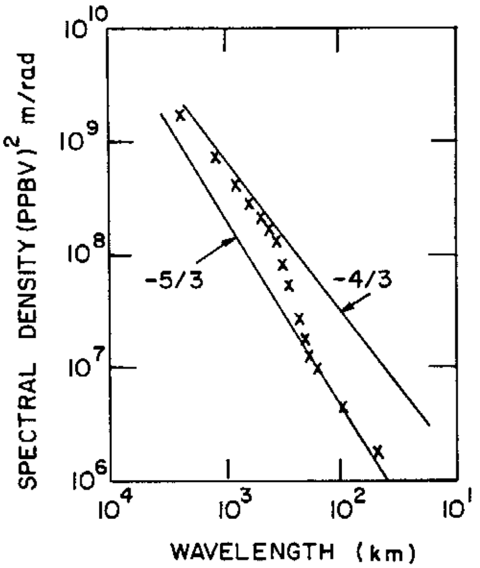

Fig. 10. Spectra of ozone in stratosphere. Adapted from Gage and Nastrom (1986c).

While the energy spectra are seen to follow clearly the relation 18 [7], the temperature spectrum *does not follow* the power law 19 with the exponent "- 4/3" (see also Gage and Nastrom (1986b)) but rather the power law *with the same exponent "-7/3"*. Note, that in Bershadskii et al. (1993b) it has been erroneously asserted that the expression of type 18 for E_c (!) can be obtained employing as a governing parameter ζ. Before addressing this additional "anomaly" we show three results in which *a clear range with the exponent "-4/3" does appear.* The first result presented at Fig. 9 shows the low frequency part of fluctuations of temperature obtained from a month-long series of radiosonde soundings taken over Kharkov, USSR, in July 1966 (Vinichenko and Dutton , 1969).

The second result regarding the spectrum of ozone in the stratosphere in the GASP (Gage and Nastrom , 1986c) is shown in Fig. 10. An indication of similar behaviour of spectra of carbon monoxide can be seen too (Gage and Nastrom , 1986c).

The third result has been obtained for temperature fluctuations in a totally different situation: in a laboratory flow past a circular cylinder at a distance about 100 diameters downstream of the cylinder on the wake centerline (Sreenivasan , 1991). It is shown in Fig. 11 and exhibits a slope "-4/3" over more than 1.5 decades (in Prasad and Sreenivasan (1990) this slope was observed over more than two decades).

An important feature of this last result is that the "-4/3" **scaling at the low-wave number end extends to scales substantially larger than** L (Sreenivasan , 1991) (L - is the velocity correlation, or 'integral', length scale), i.e. this result is consistent both with the fact that the flow in the wake of a circular cylinder is dominated by large quasi-two-dimensional structures and with the use of ζ as a governing parameter as above. It should be noted that Sreenivasan (1991) ascribes the '-4/3' exponent to insufficiently large Reynolds number. Howewer, the evidence given in Sreenivasan (1991) for larger Reynolds numbers is not of such quality as in Fig. 11 and seems to be inconclusive. A slope very close to '-4/3' was recently obtained in Jaesh et al. (1994) for temperature fluctuations in a grid turbulent flow when the temperature fluctuations were introduced by fine wires placed downstream from the grid in a *parallel array* (the spectra were different when the temperature fluctuations were introduced from a heated grid (Warhaft and Lumley , 1978) or by a *toaster* (Jaesh et al. , 1994)). In this last case the velocity spectrum had the *same* slope. Therefore it seems that the results of Jaesh et al. (1994) cannot be explained using the argument based on the helicity invariant only and the question remains open.

Let us return to the power law for $E_c(k)$ with the exponent "-7/3". *It can be obtained via dimensional arguments too* taking instead of ζ the following governing parameter

$$\eta = \langle | \frac{\partial \epsilon}{\partial z} | \rangle, \tag{20}$$

[7] Other examples are given in Bershadskii et al. (1993b). In fact, velocity spectra with the slope close to '-7/3' were observed earlier (Pao and Goldburg (1969), Monin and Ozmidov (1985)). For example, in their Fig. 13.1 Monin and Ozmidov (1985) compiled data of different authors on one-dimensional spectra of large-scale meteorological fields. It should be emphasized thta the slope '-7/3' is much closer to the data than the slope '-3' drawn in this figure.

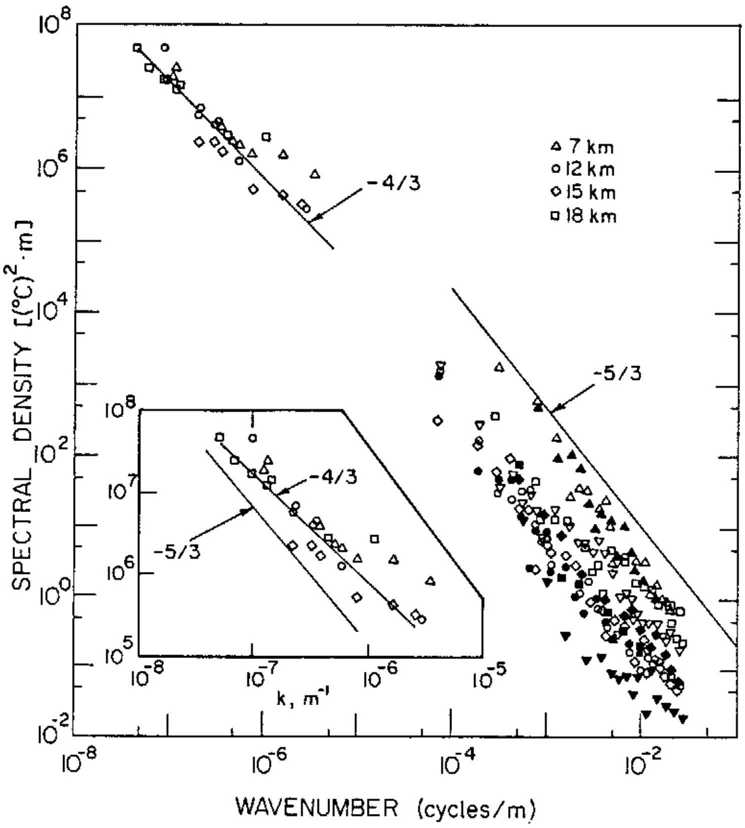

Fig. 9. Temperature spectra in the free atmosphere. The close up of the low frequency part is shown in the inset. Adapted from Vinichenko & Dutton (1969).

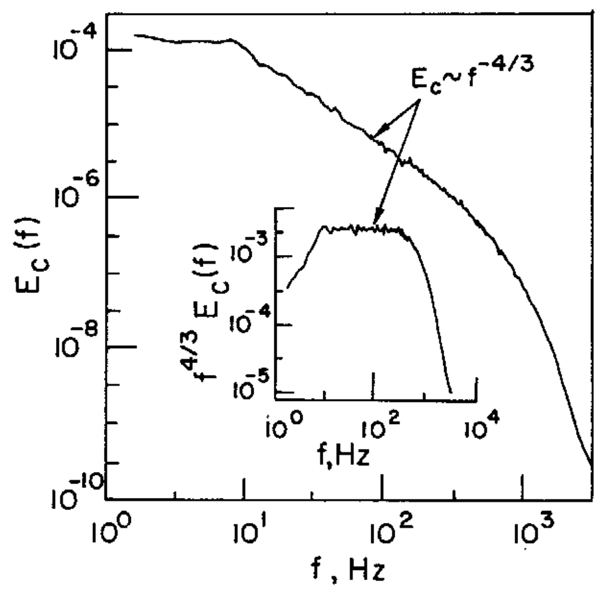

Fig. 11. Spectral density of temperature fluctuations in the wake of heated cylinder of circular cross section. Adapted from Sreenivasan (1991).

which has the meaning of average variation of "two-dimensional" dissipation in the direction z of slow variation of flow properties. It follows in this case that

$$E_c(k) \sim \langle | \frac{\partial N}{\partial z} | \rangle \eta^{-1/3} k^{-7/3}. \tag{21}$$

The parameter η has the same dimensionality as ζ and therefore the energy spectrum in the form 18 can be obtained empoying η as a governing parameter too (Branover et al. , 1993). The difference in spectra of $E_c(k)$ in 19 and 21 arises due to different contributions of $N = | dc^2/dt |$. One of the possible ways of resolving the issue of the parameter η versus ζ is that in the GASP data the *potential temperature is not really passive*, since in this particular case *the magnitude and shape of potential temperature spectrum are determined by the same dynamics that govern the velocity spectra* (Gage and Nastrom , 1986b), whereas ozone and carbon monoxide are passive (Gage and Nastrom , 1986c). It should be also emphasized, that in the laboratory experiments mentioned above the temperature (Sreenivasan , 1991) and the dye (Prasad and Sreenivasan , 1990) *were passive* and obeyed 19. Therefore, it seems that the parameter ζ is the relevant one (see the last section for discussion).

Let us look now at other parameters related to turbulent diffusion.

The turbulent diffusion coefficient $\mathcal{K}$ for quasi-two-dimensional t urbulence can also be obtained via dimensional arguments in the following form

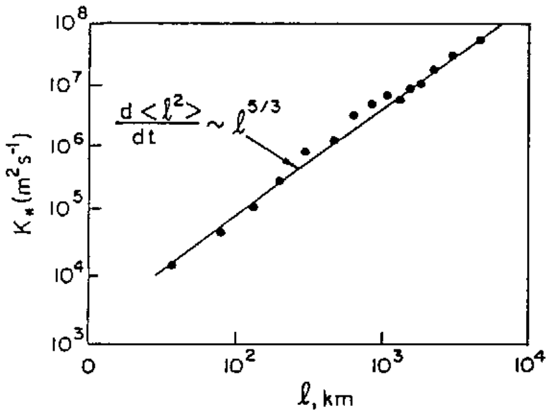

Fig. 12. Mean square relative velocity of balloon pairs in stratosphere. Adapted from Morel and Larcheveque (1974).

$$\mathcal{K} \sim d(\langle \ell^2 \rangle)/dt \sim \zeta^{1/3}\, \ell^{5/3}, \tag{22}$$

where ℓ is the characteristic scale of the cloud of the passive substance.

The expression 22 is different both from the case of 3-D turbulence with

$$\mathcal{K} \sim \ell^{4/3}, \tag{23}$$

and from the case of purely two-dimensional turbulence with

$$\mathcal{K} \sim \ell^{2} \tag{24}$$

in the range of enstrophy transfer (i.e. $E(k) \sim k^{-3}$) (in the range of energy transfer in 2D case $\mathcal{K} \sim \ell^{4/3}$).

In a similar way an expression for the mean square relative velocity can be obtained

$$\langle (d\ell/dt)^2 \rangle \sim \zeta^{2/3}\, \ell^{4/3}. \tag{25}$$

The relations 22 and 25 are in agreement with the results of experiments on diffusion of passive scalar in the lower stratosphere (Morel and Larcheveque , 1974) as can be seen from Fig. 12 and Fig. 13.

Similar behaviour was observed in a laboratory experiment on turbulence in a rotating fluid (Mory and Hopfinger , 1986) (Fig. 14).

The situation considered in this section (with *spontaneous* generation of helicity in Q2D turbulence) is different from the case of turbulence with *extrinsically* imposed mean helicity, which can have considerable influence on the transport properties (see Cattaneo et al. (1988), Chechkin et al. (1993), Drummond et al. (1984), Moffatt (1983) and references therein).

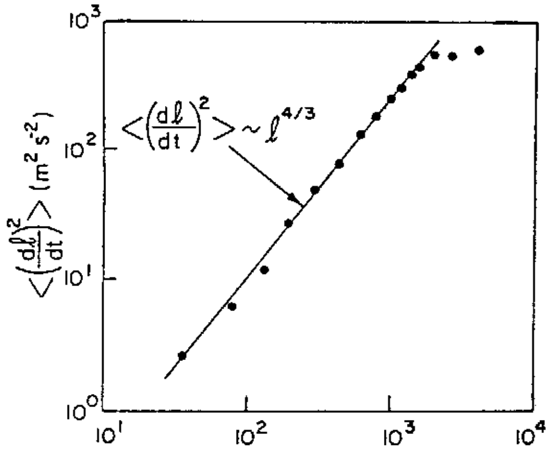

Fig. 13. Mean square relative velocity of balloon pairs in stratosphere. Adapted from Morel and Larcheveque (1974).

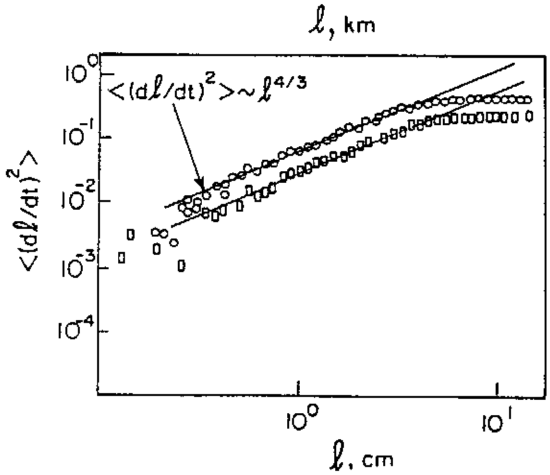

Fig. 14. Mean square relative velocity of particle pairs in a rotating fluid. Adapted from Mory and Hopfinger (1986).

5 On fractal properties of turbulent diffusion of passsive scalar

I. It can be expected that fractal properties of turbulent diffusion[8] should be qualitatively different for

[8] It is noteworthy that Corrsin (1959) gave a beautiful illustration of fractal nature of turbulent diffusion, see his Fig. 9.

three-dimensional (Kolmogorov) and Q2D turbulence discussed in section 4.

In order to illustrate this difference consider the expansion of a cloud of turbulent fluid, which is symmetric in the mean and its cross section passing through its geometric center (again in the mean) with an effective radius $R(t)$. Approximating the perimeter of this cross section by a broken line consisting of sections of length r the number N of such sections

$$N \sim (r/R)^{-D_p}, \tag{26}$$

where D_p is the *fractal dimension* of the perimeter of the cloud cross section which is related to the fractal dimension of the cloud surface by the simple relation: $D_\sigma = D_p + 1$.

In order to relate the fractal dimension with the exponent in the power spectrum of passive scalar let us find the effective rate of increase of the area of the cloud cross section ds/dt. Using the simple relation

$$ds = r(\delta u_r dt)N, \tag{27}$$

where δu_r is the velocity of the section r normal to it (Townsend , 1966), and (4) it follows that

$$ds/dt \sim r^{1-D_p} \delta u_r R^{D_p}. \tag{28}$$

The velocity δu_r can be estimated via the well known relation (Monin and Yaglom , 1971, 1975)

$$\delta u_r \sim r^\alpha, \quad \text{where} \quad \alpha = (\gamma - 1)/2, \quad E(k) \sim k^{-\gamma}, \tag{29}$$

and the relation 28 becomes[9]

$$ds/dt \sim r^{1-D_p+\alpha}. \tag{30}$$

Finally, since the rate of increase of area of the cloud cross section *is independent of* r (which has been used for its approximation) it follows from 29 that

$$D_p = 1 + \alpha = (1+\gamma)/2. \tag{31}$$

Let us look at two important cases:

◇ – Kolmogorov turbulence (which is 3D). In this case $\gamma = 5/3$ and $D_p = 4/3$.

◇ – Quasi-two-dimensional turbulence. In this case (see equation 18) $\gamma = 7/3$.

These numbers are in good agreement with measurements of area $\mathcal{A}$ versus perimeter $\mathcal{P}$ of rain and cloud areas, determined from radar and satellite data (Lovejoy , 1982), shown in Fig. 15. We have drawn straight lines corresponding to $D_p = 4/3$ and $D_p = 5/3$ using the relation (Mandelbrot , 1982) $\mathcal{A} \sim \mathcal{P}^{2/D_p}$. Gifford (1989) recognized that *two* values of D_p are better characterizing the data of Lovejoy (1982) than the single value $D_p = 1.35$, reported by Lovejoy for all points.

[9]It should be empasized that the relation 30 is the simplest (average) of this kind. It becomes more complicated after taking into account different singularities of the turbulent field.

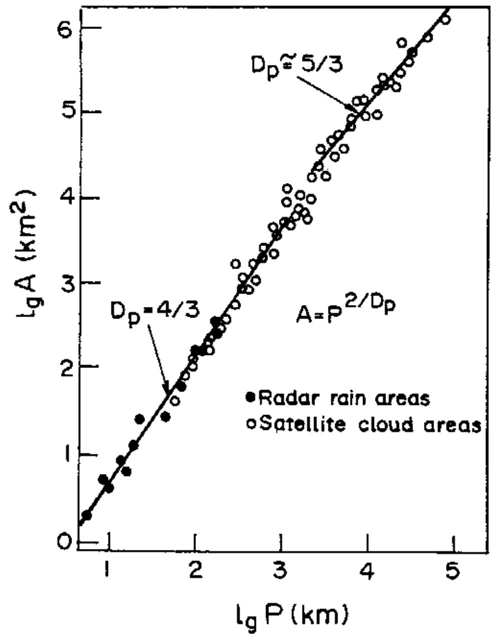

Fig. 15. Area $\mathcal{A}$ versus perimeter $\mathcal{P}$ for cloud and rain areas Adapted from Lovejoy (1982).

II. It is a common assumption in turbulence research that turbulent dissipation is well represented by a single squared velocity derivative (dissipation surrogate). However, *except of their mean values* other properties of these two quantities are *different even in homogeneous and quasi-isotropic flows*. This problem has been posed by Gibson and Masiello (1972) (see also Sreenivasan et al. (1977), Tsinober et al. (1992)). In case of a passive scalar it is much more difficult to realise the reasons for such a difference. However, there exist clear indications that there is a considerable difference between fractal and multifractal properties of the dissipation rate of passive scalar and its surrogate. For example it has been shown (Bershadskii and Tsinober , 1993) that the fractal dimension (Kolmogorov capacity) of the carrier of the rate of dissipation of passive scalar is equal to 3, while its' value for an individual squared derivative is 5/3 (D_∇). This result is in good agreement with the one obtained from a recent simulation of turbulent dispersion (Stiassnie et al. , 1993). Some results adapted from Stiassnie et al. (1993) are shown in Fig. 16 and Fig. 17. It is seen that the fractal dimension D_∇ reduces from 3 at the initial moment to about 5/3 at $t = 1sec$ (Fig. 16). It has been pointed out correctly by one of the referees that the 'evolution' of D_∇ shown in Fig. 16 is linked to multifractality. At the same time the dimension of the perimeter of the projection of the cloud surface increases from 1 at $t = 0$ to about 4/3 at $t \geq 0.5sec$ (Fig. 17).

Similar results were obtained for the dimension of the perimeter of the large-scale cloud structures in the tropical zone ($\cong 1.34$) (Baryshnikova et al. , 1989), clouds over Indian region (≅ 1.3) (Jain , 1989), (Jayanthi et al. , 1990), in measurements of the Chernobyl spot of the radionuclides contamination (≅ 1.37) (Bar'yakhtar et al. , 1993) and in a laboratory smoke plume (≅ 1.43) (Praskovsky et al. , 1993).

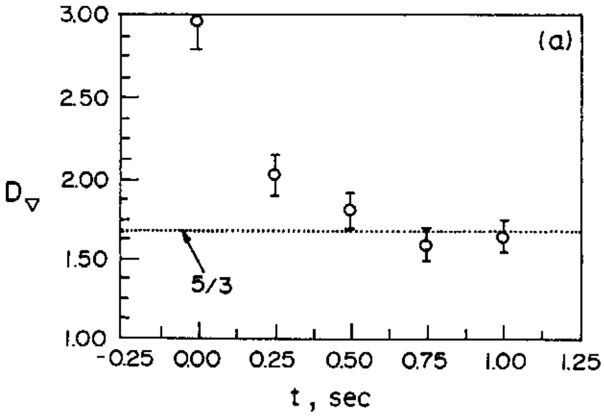

Fig. 16. Evolution of the fractal dimension $D_{\nabla'}$ of a cloud of tracer particles. Adapted from Stiasnie et al. (1993).

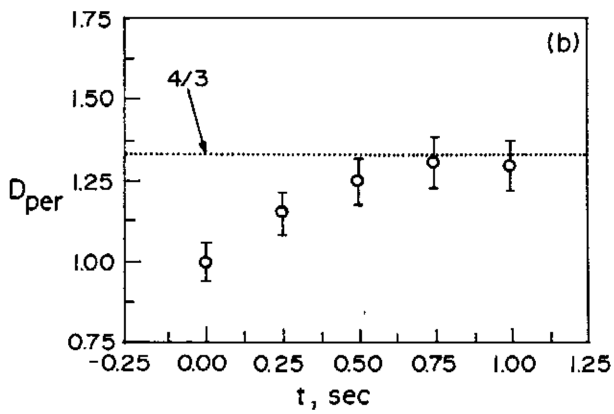

Fig. 17. Evolution of the fractal dimension of the perimeter of the cloud suface projection. Adapted from Stiasnie et al. (1993).

6 Summary, discussion and some open problems

Summarizing we would like first to reiterate the main points of this communication.

$\mathcal{A}$. It is argued that regions with large fluctuations of turbulent energy are characterized by strong anisotropy and a local cascade of angular momentum, i.e. of a quantity of the type of Loytsianskii's invariant. These arguments - which have been supported by laboratory and numerical data on asymptotic properties of higher order intermittency exponents of turbulent energy - have been used for the analysis of diffusion of a puff of passive scalar. The result is a scaling law for the turbulent diffusivity $\mathcal{K} \sim \ell^{4/5}$, where ℓ is the characteristic scale of the puff. This relation appears to be in good agreement with many observations (Bershadskii et al. , 1994).

$\mathcal{B}$. It is claimed that Kolmogorov turbulence is critical in respect to the localization effects of subregions with large helicity (*helical fractons*) and that the Kolmogorov cascade is renormalized in the *helical fractons*. The *quantitaive* consequences of such a renormalization have been confirmed by the analysis of the asymptotic behaviour of the higher order intermittency exponents of the field of helicity, obtained in three different turbulent laboratory flows (grid, boudary layer and jet). These results lead to a scaling law $\mathcal{K} \sim \ell^{8/7}$ in which the turbulent diffusion is controlled by helical fractons. This scaling law is in good agreement with a variety of observations in troposphere and in the ocean (Bershadskii et al. , 1993a).

$\mathcal{C}$. It is shown that the asymptotic properties of the higher order intermittency exponents of turbulent disspation and other geometrical invariants in quasi-two-dimensional turbulence (arising as a result of helical instability of purely two- dimensional turbulence) are controlled by a global quasi-two-dimensional cascade of helicity (Bershadskii et al. , 1993b). Again this is confirmed by the results of laboratory modelling of quasi-two-dimensional turbulence (Branover et al. , 1993). In this case the scaling law for the turbulent diffusivity is $\mathcal{K} \sim \ell^{5/3}$. This scaling law and the corresponding fractal and spectral scaling relations are observed in the large scale stratospheric turbulence.

Thus local spontaneous breaking of isotropy of turbulent flow results in anomalous scaling laws for turbulent diffusion (as compared to the scaling law of Richardson) *and are observed, as a rule, in different atmospheric layers from the atmospheric boundary layer (ABL) to the* *stratosphere. The breaking of rotational symmetry is important in the ABL, whereas reflexinonal symmetry breaking is dominating in the troposphere locally and in the stratosphere globally.*

As has been already mentioned in the abstract the above results are of speculative nature (mainly due to use of dimensional arguments and scalings) [10] and leave several important questions open. Some of these questions are discussed below.

An important criterion of validity of results obtained via dimensional arguments is that *different* characteristics of the flow obtained in such a way should be consistent with the *same* governing parameter. For example, in the case when the governing parameter is $\mathcal{L}$ (see section 2) the turbulent energy spectrum should have the form

$$E_u(k) \sim \mathcal{L}^{2/3}\ k^{-3/5}, \tag{32}$$

and the spectrum for $E_c(k)$

$$E_c(k) \sim \langle N \rangle \mathcal{L}^{-1/3}\ k^{-11/5}, \tag{33}$$

The available evidence does not allow to make a definite judgement about the existence of spectra 32 and 33. However, it seems that such spectra can be observed in appropriate conditions. Indeed, a spectrum $E_c(k)$) with the exponent "-11/5" was observed over almost two decades in the low wave number region in the experiments in the coastal region of the Baltic sea (Ozmidov et al. , 1971), Fig. 18. Monin and Ozmidov (1978) see the reason for such a spectrum in the possibilty of energy supply over (almost) the whole range of scales (cf. section 2). It is worth to note that the "-11/5" spectrum is obtained from totally different considerations for vertically stratified wind turbulence (Bogliano-Obukhov scaling).

In case when the governing parameter is $\tilde{\epsilon}$ (section 3) the situation is more serious, since the energy spectrum in this case takes the form

$$E_u(k) \sim \tilde{\epsilon}^{\ 6/7}\ k^{\ -9/7}, \tag{34}$$

and is ***not compatible with the existing experimental evidence***. It has been claimed in Bershadskii et al. (1993a) that in this particular case *the energy spectrum can be not compatible with 15 in the generally accepted sense due to complicated structure of helical fractons which are able to trap and detain the passive scalar within their interior*. This claim, of course, requires further elaboration, but even if it is true the spectrum of $E_c(k)$

$$E_c(k) \sim \langle N \rangle \tilde{\epsilon}^{-3/7}\ k^{-13/7}, \tag{35}$$

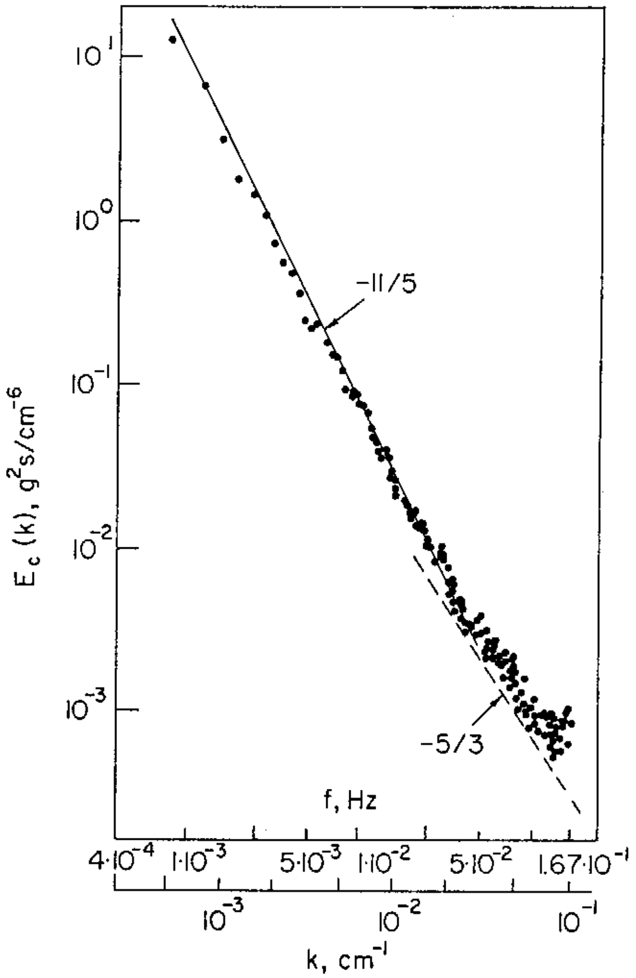

Fig. 18. Spectrum of dye concentration in experiments with continuous source (Ozmidov et al., 1971). Adapted from Monin and Ozmidov (1978).

[10] Citing R. K. Kraichnan - *The wonderfull thing about scaling is that you can get everything right without understanding anything* (Kadanoff , 1990).

Also the citation from R. E. Normal should be seen as a warning: *It is increasingly clear that deterministic chaos and universal scaling theories can explain everything* (Normal , 1993).

should be compatible with the corresponding governing parameter. Again there exist no firm evidence on the existence of the spectrum 35.

Finally, in case when the governing parameter is ζ (see section 4, equation 17) there is an alternative parameter η (Branover et al. , 1993) (equation 20). As has been poined in section 4 it is more likely that the relevant parameter is ζ as having clear physical meaning and compatible with the existing experimental evidence, though rather limited in the case of a passive scalar. In this respect the results of Branover et al. (1993) should be seen as supporting the first choise, especially in view of the results for $E_c(k)$ obtained in Vinichenko and Dutton (1969), Gage and Nastrom (1986b) and Sreenivasan (1991), Prasad and Sreenivasan (1990). Still, the possibility of (co)existence of both situations cannot be excluded totally and the issue remains open including the question about possible relation between ζ and η. A trivial (but almost useless) answer to the last question follows again from the very dimensional argument, i.e.

$$\zeta = \langle | \, dh/dt \, | \rangle \sim \eta = \langle | \, \frac{\partial \epsilon}{\partial z} \, | \rangle, \tag{36}$$

the physical meaning of which (if such can be found) is not clear.

It is noteworthy that the '$-7/3$' turbulent energy spectrum can be obtained from totally different considerations as an exact solution of the kinetic equation for inertial-gravity waves (Falkovich and Medvedev (1992), Falkovich (1992)) and from the so called 2.5-dimensional averaged equations for rotating fluid (Mahalov , 1993). The relation of thise approaches to the discussed above properties of Q2D turbulence is not clear yet.

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