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Heavy ion acceleration at parallel shocks

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Abstract. A study of alpha particle acceleration at parallel shock due to an interaction with Alfvén waves self-consistently excited in both upstream and downstream regions was conducted using a scale-separation model (Galinsky and Shevchenko, 2000, 2007). The model uses conservation laws and resonance conditions to find where waves will be generated or damped and hence where particles will be pitch-angle scattered. It considers the total distribution function (for the bulk plasma and high energy tail), so no standard assumptions (e.g. seed populations, or some ad-hoc escape rate of accelerated particles) are required. The heavy ion scattering on hydromagnetic turbulence generated by both protons and ions themselves is considered. The contribution of alpha particles to turbulence generation is important because of their relatively large mass-loading parameter $P_\alpha = n_\alpha m_\alpha / n_\mathrm{p} m_\mathrm{p}$ (m_p, n_p and m_α, n_α are proton and alpha particle mass and density) that defines efficiency of wave excitation. The energy spectra of alpha particles are found and compared with those obtained in test particle approximation.

1 Introduction

An explanation of the main characteristics of accelerated particles in astrophysical plasmas became possible due to an idea of diffusive shock acceleration (DSA) at quasi-parallel shocks (Krymsky, 1977; Axford et al., 1977; Bell, 1978a, b; Blandford and Ostriker, 1978). The DSA mechanism was first developed for astrophysical shocks following the suggestion that charged particles are accelerated through first order Fermi acceleration by clouds of Alfvén waves excited on both sides of the shock wave. Soon after Lee (1983), following the work of Skilling (1975) and Bell (1978a, b), developed the quasi-linear model that describes self-consistently the ion acceleration and the wave excitation at interplanetary shocks. Lee (1983) (see also Gordon et al., 1999) assumed, that some seed population of protons is injected at the shock front into the upstream plasma and developed a mechanism to investigate the dynamics of only this population due to interaction with waves. As a result of the cyclotron instability (Sagdeev and Shafranov, 1961), Alfvén waves are excited and scatter some particles to the downstream region creating the conditions for the first order Fermi mechanism. The main assumption in this approach, similarly to (Skilling, 1975; Bell, 1978a, b), is the near isotropicity of the distribution function. This assumption can be justified only if the time scale of particle pitch angle scattering due to cyclotron interaction is the shortest time scale of all physically relevant processes. Then the pitch-angle averaged distribution function can be found as a solution of the so-called convection-diffusion equation with right-hand side describing the ion source term (see e.g. Eq. 2.1 in Malkov and Drury, 2001). To relate this solution to observations, the flux density of the seed particles in the source term should be obtained from either direct or indirect shock measurements.

In spite of successful explanation by DSA-based analytical and numerical models (Lee, 1983; Jokipi, 1987; Gordon et al., 1999; Zank et al., 2000; Li et al., 2003, 2005; Giacolone and Kota, 2006) of many important features of the acceleration process, for example power spectra of accelerated particles, dependence of accelerated particles on the distance from the shock front and many others, there are important questions that remain open for a long time.

The possibility of thermal plasma to provide a source of the accelerated protons has been discussed for a long time. There was also considerable evidence obtained both from observations and simulations, that shocks can directly accelerate ambient thermal particles (see reviews by

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Drury (1983) and by Jones and Ellison (1991) and references therein). It was also shown by Malkov & Völk (1995) that the wave fields required for proton acceleration can be excited by a beam of the downstream tail protons injected into the upstream region. This mechanism cannot be incorporated in the macroscopic acceleration picture based on the theoretical models derived from DSA approach. The DSA-based theoretical models do not follow the dynamics of thermal plasma and hence they cannot describe the process of proton acceleration from the thermal plasma.

Another important problem is that the DSA models do not include the back reaction of the accelerated particles on the shock wave that can limit the process of acceleration (although some models were able to limit extent of acceleration by introducing wave damping; Völk et al., 1988). And again, this shortcoming is connected with limitations of DSA approach that evaluates only the dynamics of the high energetic tail of particle distribution. Detailed discussions of DSA-based models can be found in reviews (see e.g. Drury, 1983; Jones and Ellison, 1991; Malkov and Drury, 2001) and references therein).

Recently, a new theoretical approach and numerical model of proton acceleration at quasi-parallel shocks based on energy balance between waves and resonant particles has been proposed in Galinsky and Shevchenko (2007) (we will hereafter refer to this paper as "Paper 1"). This model automatically includes both the thermal plasma injection scenario and modification of the shock structure due to the reaction of the accelerated particles. Similarly to DSA-based models (Krymsky, 1977; Axford et al., 1977; Bell, 1978a, b; Blanford and Ostriker, 1978), the new model is one-dimensional where waves propagate parallel or anti-parallel to the magnetic field. A quasi-linear approach was used in Paper 1 for description of wave-particle interaction, similarly to the analytical consideration by Lee (1982) and Gordon et al. (1999), and the resonant wave-particle interactions were assumed to be the fastest processes in the particle acceleration problem.

However, in Paper 1, in contrast to Lee (1982), Gordon et al. (1999), plasma protons were not divided into two classes of resonant protons and thermal plasma. The evolution of the entire gyro-phase averaged proton distribution function was analyzed. Stability of the velocity distribution in the total interval of possible parallel velocities $v_\parallel$ was investigated for each distance from the shock front at each time step. Using a quasi-linear approach, the energy exchange between particles and waves for each interval of resonant parallel velocities $\Delta v_\parallel$ was analyzed to find a new velocity distribution function at the interval $\Delta v_\parallel$ as well as the corresponding wave power spectrum. As a result, the dynamics of the entire particle distribution and the wave power spectrum was studied as function of time and distance from the shock front. Paper 1 demonstrated for the first time how the problem of particle acceleration from the thermal distribution can be included in a natural way in a macroscopic model of shock acceleration. The model constructed in Paper 1 takes into account the pressure of accelerated particles that can decelerate the upstream flow of the solar wind and modify the shock wave structure.

Using this model, Paper 1 studies proton acceleration in the case of a shock wave propagating parallel to the ambient magnetic field. These results showed agreement with diffusive shock acceleration (DSA) models in prediction of power spectra for accelerated particles in upstream region. However, this study has also revealed the presence of a spectral break in the high-energy part of the particle spectra. It was also found that in the downstream region close to the shock front, strong diffusion over perpendicular energy takes place for particles with small absolute values of parallel velocity $v_\parallel$ in the shock reference frame. These particles were quite likely to cross the shock interface multiple times and interact with Alfvén waves at cyclotron resonance conditions that are different for waves in upstream and downstream regions. As a result, particle were not only pitch angle scattered but energized as well, mainly in a perpendicular direction.

One of the most interesting problems in the shock acceleration is the problem of heavy ion acceleration. Up to now this problem was considered using a test particle approximation assuming that the waves needed for ion acceleration are excited by protons and that the wave excitation by heavy ions is not sufficient, and therefore, neglected. Using a test particle approach, various DSA models were able to explain results of observations of the abundance of various heavy ions in heliosphere with reasonable success. In brief, the analysis showed a consistent dependence of heavy ion abundances on A/Z (the atomic weight to charge ratio). The only deviation from this A/Z ordering has been found in alpha particle behavior and in the behavior of protons themselves (Meyer, 1985; Mason, 1987). Attempts to explain this deviation by DSA models required artificially high injection rates in order for the results to be at least qualitatively consistent with observations (Tylka et al., 1999; Ng et al., 1999). We would like to note that the efficiency of the wave excitation in the solar wind is defined by the so-called mass loading parameter $P_\alpha = n_\alpha m_\alpha / n_\mathrm{p} m_\mathrm{p}$ (see Eq. 17) that has a reasonably high value in the case of alpha particles. So it is important to understand how much a self-consistent approach to heavy ion acceleration can change the efficiency of ion acceleration.

In this paper we will study alpha particle acceleration at a shock wave that propagates parallel to the interplanetary magnetic field.

2 Equations and algorithm of numerical solution

We consider a planar shock wave moving along the z-axis with the +z direction parallel to the magnetic field, and we will work in the wave front system of reference.

We assume that initially the shock front at $z=z_0$ divides the upstream and downstream plasmas with Maxwellian distributions with parameters that satisfy Rankine-Hugoniot boundary conditions (see e.g. Jones and Ellison, 1991):

$$\left[\boldsymbol{u}\cdot\hat{\boldsymbol{n}}\left\{\frac{\gamma}{\gamma-1}P+\frac{1}{2}\rho u^2+\frac{B^2}{4\pi}\right\}-\frac{\left(\boldsymbol{B}\cdot\hat{\boldsymbol{n}}\right)\left(\boldsymbol{B}\cdot\boldsymbol{u}\right)}{4\pi}\right]_{\rm d}^{\rm up}=0$$

$$\left[\rho\boldsymbol{u}\left(\boldsymbol{u}\cdot\hat{\boldsymbol{n}}\right)+\left(P+B^2/8\pi\right)\hat{\boldsymbol{n}}-\left(\boldsymbol{B}\cdot\hat{\boldsymbol{n}}\right)\boldsymbol{B}/4\pi\right]_{\rm d}^{\rm up}=0 \qquad (1)$$

$$\left[\boldsymbol{B}\cdot\hat{\boldsymbol{n}}\right]_{\rm d}^{\rm up}=0$$

$$\left[\hat{\boldsymbol{n}}\times(\boldsymbol{u}\times\boldsymbol{B})\right]_{\rm d}^{\rm up}=0$$

Here ρ, $\boldsymbol{u}$, and P are the zero, first and second moments of the proton distribution function, $\hat{\boldsymbol{n}}$ is a unit vector in the direction of the shock normal (negative z-direction), $[f(z)]_{\rm d}^{\rm up}$ is the difference between initial upstream (up) and downstream (d) values of $f(z)$ at the wave front. We would like to note that we need boundary conditions only at the time $t=0$. Similarly to Paper 1, we solve the spatial-temporal problem of plasma-wave dynamics by considering the upstream and downstream plasmas as one plasma with inhomogeneous parameters.

As in Lee (1983), Gordon et al. (1990 and in Paper 1, we do not use any external forces and rely only on resonant wave-particle interaction self-consistently included in the model to excite Alfvén waves and accelerate particles. We introduce the wave action

$$W^{\leftrightarrow}(t,z,\omega_k)=|B_k^{\leftrightarrow}|^2/8\pi\omega_k$$

which describes the wave packets propagating parallel ($\rightarrow$) and anti-parallel ($\leftarrow$) to the external magnetic field in a medium with varying parameters that are calculated from moments of the particle distribution function. $|B_k|^2$ is the spectral density of the wave magnetic field. The resonance conditions for interaction with such waves have the form

$$\omega k-k v_{||}\mp\omega_{\rm c}=0 \qquad (2)$$

Here ω_k and k are the frequency and wave number of Alfvén waves, $\omega_{\rm c}$ is the proton cyclotron frequency. The $\mp$ signs in Eq. (2) describe interactions with waves at normal and anomalous Doppler resonances correspondingly.

Cyclotron resonant interaction leads to pitch angle scattering of particles, so resonant protons interacting at both possible resonances (2) with each broadband packet $W^{\leftrightarrow}(t,z,\omega_k)$ diffuse along the pitch angle diffusion lines (Vedenov et al., 1962; Rowlands et al., 1966; Kennel and Engelman, 1966)

$$w^{\leftrightarrow}=v_\perp^2+\left(v_\parallel-v_{\rm ph}^{\leftrightarrow}\right)^2={\rm const} \qquad (3)$$

where $v_{\rm ph}^{\rightarrow}=v_{\rm A}$ and $v_{\rm ph}^{\leftarrow}=-v_{\rm A}$.

As a result of resonance interaction, a shell-like distribution of protons $f=f(w)$ is formed in the interval of resonant velocities v_z.

As was discussed above, we will study the dynamics of the wave excitation and particle acceleration relying on the thermal plasma as a source of so called seed population that excites waves needed for particle acceleration. Thus, similarly to Paper 1, we will analyze stability of the entire gyro-phase averaged proton distribution function. By averaging the proton kinetic equation over times larger than the characteristic period of oscillations we obtain in quasi-linear approximation the equation for so-called background distribution function of protons $f(t,z,v_{||},v_\perp)$:

$$\frac{\partial f}{\partial t}+v_\parallel\frac{\partial f}{\partial z}=QL_f\left(f,W^{\rightarrow},W^{\leftarrow}\right) \qquad (4a)$$

Equations for wave actions in quasi-linear approximation have the form:

$$\frac{\partial W^{\leftrightarrow}}{\partial t}+\frac{\partial}{\partial z}\left(v_{\rm gz}^{\leftrightarrow}W^{\leftrightarrow}\right)=QL_W^{\leftrightarrow}\left(f,W^{\leftrightarrow}\right) \qquad (5a)$$

Here $v_{\rm gz}^{\leftrightarrow}$ is z-component of the group velocity for waves propagating in parallel and anti-parallel directions.

Right-hand-side terms in Eqs. (4)–(5) are quasi-linear operators that have the following form in plasma reference frame:

$$\begin{aligned} QL_f &= \frac{2\pi e^2}{m_p^2c^2} \\ &\cdot\ \sum_{\mp}\left\{\hat{L}\left[v_\perp^2\frac{\omega_k W_k^{\rightarrow}}{\left|v_\parallel-\partial\omega/\partial k\right|}\hat{L}f\right]\right|_{v_\parallel=v_{\rm A}(\mp\omega_{\rm c}+\omega)/\omega} \\ &+\ \left.\hat{L}\left[v_\perp^2\frac{\omega_k W_k^{\leftarrow}}{\left|v_{||}+\partial\omega/\partial k\right|}\hat{L}f\right]\right|_{v_\parallel=-v_{\rm A}(\mp\omega_{\rm c}+\omega)/\omega}\Bigg\} \qquad (4b)\end{aligned}$$

$$QL_W^{\leftrightarrow}=\frac{v_{\rm ph}^{\leftrightarrow}}{v_{\rm A}}\frac{\pi^2}{n_0}\frac{\omega_{\rm c}^2}{\omega_k}W_k^{\leftrightarrow}\int v_\perp^3\hat{L}f\,dv_\perp\Big|_{v_\parallel=v_{\rm ph}^{\leftrightarrow}(\mp\omega_{\rm c}+\omega)/\omega} \qquad (5b)$$

$$\hat{L}\psi=\frac{\partial\psi}{\partial v_\parallel}+\frac{\omega_k-k_{||}v_{||}}{v_\perp k_{||}}\frac{\partial\psi}{\partial v_\perp}$$

The sum in Eq. (4b) takes into account interactions at normal and anomalous Doppler resonances. Thus, the quasi-linear term in the right-hand side of Eqs. (4a) and (4b) takes into account all four possible interactions with MHD waves propagating in both directions along the magnetic field.

The Eqs. (4)–(5) describe processes of the wave excitation and particle acceleration in the system under consideration. Since these equations are non-stationary and non-homogeneous nonlinear equations, we solve them numerically. The region over z with size L where processes of the wave excitation and particle acceleration take place stretches in both directions from the shock front. The size L of the region is chosen to be much larger than the characteristic scale of quasi-linear relaxation. We divide the region L into small intervals with locations

$$z_i, i = 1, 2, 3, \ldots N_z \tag{6}$$

and will solve Eqs. (4)–(5) at each interval for each time step. To do this, we introduce the wave spectrum with a wide band of possible wave numbers

$$k_i, i = 0, 1, \ldots N_k \tag{7}$$

to assure that resonant cyclotron interaction samples all possible intervals of resonant velocities $v_\parallel$ on the proton distribution function.

Similarly to Paper 1, scale-separation techniques will be used to numerically solve Eqs. (4)–(5) by assuming that characteristic temporal and spatial scales of pitch-angle diffusion (microscopic scales) are smaller than the time step and size of each spatial interval (macroscopic scales). That means that steady state of the plasma-wave system is developed at each time step at any distance from the shock front. The steady state at some interval of resonant velocities is achieved in two cases: (i) when the pitch-angle averaged distribution function is formed $\hat{L} f = 0$ or (ii) when the resonant waves are totally absorbed $W_k = 0$ at a considered interval of resonant velocities. Thus, at the end of each time step, the right-hand-side terms in Eqs. (4)–(5) are equal to zero.

For the purpose of calculating the input from the resonant particles to the waves and vice-versa, we divide the entire region of all possible parallel and perpendicular velocities into small intervals with grid locations

$$v_\parallel^i, i = 1, 2, 3, \ldots N_{v_\parallel} \tag{8}$$

$$v_\perp^i, i = 1, 2, 3, \ldots N_{v_\perp} \tag{9}$$

To calculate energy transfer from waves to resonant particles and vice-versa, we need to find the free energy available in the particle distribution for each resonant velocity interval. The amount of free energy available in n-th interval of resonant velocities $v_\parallel \in [v_\parallel^n - \delta v_\parallel, v_\parallel^n + \delta v_\parallel]$ is defined by a difference between the energy contained in the current particle distribution function (PDF) and the energy contained in the pitch-angle scattered PDF both taken over the resonance interval. We find the pitch-angle average function at each time step using conservation of the proton number along diffusion lines (3) (see Paper 1):

$$\langle f\left(t, z, v_\parallel, v_\perp\right)\rangle^{\leftrightarrow} = \int_{S_n^{\leftrightarrow}} f\left(t, z, w^{\leftrightarrow}, \alpha - \alpha'\right) d\alpha' \Big/ \int_{S_n^{\leftrightarrow}} d\alpha' \tag{10}$$

where the ξ coordinate is directed along lines $w^{\leftrightarrow} = \mathrm{const}$ and $S_n^{\leftrightarrow}$ is the n-th interval of resonant velocities for waves propagating parallel ($\rightarrow$) or anti-parallel ($\leftarrow$) to the external magnetic field $v_\parallel \in [v_\parallel^n - \delta v_\parallel, v_\parallel^n + \delta v_\parallel]$.

The amount of particle free energy for each resonance region is obtained as a variation of the proton kinetic energy in final and initial states in a frame of reference where the bulk of the plasma is at rest:

$$\Delta F_m^{\leftrightarrow}(t, z) = \frac{m_\mathrm{p}}{2} \int_{S_m^{\pm}} \left[\langle f\left(t, z, v_\parallel, v_\perp\right)\rangle^{\leftrightarrow} - f\left(t, z, v_\parallel, v_\perp\right)\right] \left\{v_\perp^2 + \left[v_\parallel - u(t, z)\right]^2\right\} d\boldsymbol{v} \tag{11}$$

where $u(t, z)$ is a bulk plasma velocity.

We calculate a balance of energy between waves and particles for each resonance interval as

$$\Delta E_n^{\leftrightarrow}(t, z) = \sum_{k \in \{n\}} E^{\leftrightarrow}(t, z, \omega_k) - \Delta F_n^{\leftrightarrow} \tag{12}$$

Here $\Delta E_n^{\leftrightarrow}(t, z)$ is change of the wave energy density in n-th resonance region; $\{n\}$ represents all harmonics belonging to the n-th resonant interval that can be found from the resonance condition (2). The first term in the right-hand side of Eq. (12) is the total wave energy density in n-th resonance region at previous time step with the quantity $E^{\leftrightarrow}(t, z, \omega_k)$ defined by

$$E^{\leftrightarrow}(t, z, \omega_k) = -W_k^{\leftrightarrow} \frac{\partial D}{\partial \omega_k} \frac{\omega_k^2}{k^2 c^2} \tag{13}$$

$$D(\omega_k, k) = k^2 c^2 + \omega_\mathrm{pp}^2 \frac{\omega_k}{\omega_k - \omega_\mathrm{cp}} + \omega_\mathrm{pe}^2 \frac{\omega_k}{\omega_k + \omega_\mathrm{ce}} \tag{14a}$$

$E^{\leftrightarrow}(t, z, \omega_k)$ is the sum of potential and kinetic energy densities of the wave with frequency ω_k that is defined from equation

$$D(\omega_k, k) = 0 \tag{14b}$$

In our case the potential and kinetic wave energy densities are equal each other and $E^{\leftrightarrow}(t, z, \omega_k) = |B_k^{\leftrightarrow}|^2 / 4\pi$.

The algorithm for numeric solution of Eqs. (4)–(5) is based on discussed above energetic relationships between resonant particles and waves in a given interval of parallel velocity. In the case of unstable distribution function in this interval, instability is developed that leads to wave excitation and particle pitch angle scattering. Part of resonant particle energy is transferred to excited waves and, as a result, the energy of resonant particles with shell distribution is smaller than that of the resonant particles in the same interval of parallel velocity before the instability onset. In the case when a distribution function is stable and there are waves in a given interval of resonant velocities, the absorption of the wave energy and pitch angle scattering of resonant particles take place. The energy of resonant particles with pitch angle scattered distribution function in a given interval of parallel velocities is larger than that of the resonant particles with "initial" distribution function.

The numeric solution algorithm can be described as the following procedures:

1. It accordance with assumption that the steady state of the plasma-wave system is developed at each time step at any distance from the shock front, we update the distribution function as well as the wave spectrum at the beginning of each time step by integrating Eqs. (4)–(5) with zero right-hand sides. We obtain the new distribution function at every $z \times v_\parallel \times v_\perp$ grid location by using flux conservation and assuming one dimensional streaming of plasma in force-free environment. The waves are updated for every z_i by using conservation of their action in streaming medium with locally varying parameters.

2. The new distribution function is used to find the proton density, temperature as well as the local rest frame of reference for every z_i. Using these parameters we calculate the pitch angle diffusion lines (3).

3. By using Eq. (11), we obtain the pitch angle scattered distribution function in every resonant region $v_\parallel \in \left[v_\parallel^n - \delta v_\parallel, v_\parallel^n + \delta v_\parallel\right]$.

4. After that, using Eq. (11), we check if the energy of particles with updated distribution function is larger or smaller in comparison with that of particles with pitch angle scattered distribution function in the same resonant interval, that is if the distribution function is unstable or stable with respect to wave generation in the current resonance interval.

   (a) If it is unstable, we use the pitch angle scattered particle distribution function as a new one for the current resonance interval. We assign the available energy to the waves in this interval by using Eq. (12) and proceed to next resonant interval.

   (b) If the particle distribution function is stable but waves are present in the current resonant region, the waves will interact with the particles and transfer some or all of their energy. The new particle distribution function and the new wave level in the resonant region are found by using the same Eqs. (10)–(12).

5. Since the levels of the newly found PDF can be different in adjacent resonant intervals this locally stable PDF does not yet represent the global wave-particle equilibrium state. In order to find this global state we use aggregation procedure, that is we combine adjacent resonant intervals where the PDF has been pitch-angle scattered and use (10) to find the common pitch-angle scattered PDF. We then repeat the entire procedure (4–5) iteratively until we arrive to the (quasi)-stationary partitioning of energy between waves and particles.

In the case when initially there are waves in the plasma, we should first perform procedure 4.2 by using Eqs. (10)–(12) and proceed after that with solution of equations at the first time step using the numerical solution algorithm described above.

3 Acceleration of protons and heavy ions

To show that both protons and heavy ions are accelerated from thermal core and that the waves needed for their acceleration are excited due to cyclotron instability (Sagdeev and Shafranov, 1961), we assume that initially there is no any seed population and no waves in the system.

As was discussed before, we integrate the system of Eqs. (4)–(5) for the case of a parallel shock. We work in the shock front reference frame and initially assume that the upstream and downstream plasmas are Maxwellian with temperatures that are related by Rankine-Hugoniot boundary conditions (1). The dimensionless equations that are employed to update the distribution function and the wave spectrum at the beginning of each time step (procedure 1 in the algorithm) have the form:

$$\frac{\partial f_i}{\partial t} + v_\parallel \frac{\partial f_i}{\partial z} = QL_{f_i}\left(f_i, W^{\rightarrow}, W^{\leftarrow}\right) \quad (i = p, \alpha) \tag{15}$$

$$\frac{\partial W^{\leftrightarrow}}{\partial t} + \frac{\partial}{\partial z}\left(v_{\mathrm{gz}}^{\leftrightarrow} W^{\leftrightarrow}\right) = QL_W^{\leftrightarrow}\left(f_{\mathrm{p}}, f_\alpha, W^{\leftrightarrow}\right) \tag{16}$$

Together with Eq. (4) for a proton distribution function we now use a similar Eq. (15) for the second plasma component as well.

And the energy balance Eq. (12) can be written as

$$\begin{aligned}
&\Delta \tilde{B}_n^{\leftrightarrow 2}(t,z) = \tilde{B}_n^{\leftrightarrow 2}(t,z) \\
&-\pi\beta_{\mathrm{up}} \int_{S_n^{\leftrightarrow}} \left[\langle f_{\mathrm{p}}\left(t,z,v_\parallel,v_\perp\right)\rangle^{\leftrightarrow} - f_{\mathrm{p}}\left(t,z,v_\parallel,v_\perp\right)\right] \\
&\left\{v_\perp^2 + \left[v_\parallel - u_{\mathrm{p}}(t,z)\right]^2\right\} v_\perp dv_\perp dv_\parallel \\
&-P_\alpha\pi\beta_{\mathrm{up}} \int_{S_n^{\leftrightarrow}} \left[\langle f_\alpha\left(t,z,v_\parallel,v_\perp\right)\rangle^{\leftrightarrow} - f_\alpha\left(t,z,v_\parallel,v_\perp\right)\right] \\
&\left\{v_\perp^2 + \left[v_\parallel - u_\alpha(t,z)\right]^2\right\} v_\perp dv_\perp dv_\parallel
\end{aligned} \tag{17}$$

Here $\beta_{\mathrm{up}} = v_{\mathrm{Tup}}^2 / v_{\mathrm{Aup}}^2$, v_{Tup}, v_{Aup} are initial values of the gas kinetic to magnetic pressure ratio, the proton thermal velocity and Alfvén speed in the upstream region correspondingly and $P_\alpha = n_\alpha m_\alpha / n_{\mathrm{p}} m_{\mathrm{p}}$ is the mass-loading parameter. $\Delta \tilde{B}_n^{\leftrightarrow 2}$ in the Eq. (17) represents the change of the dimensionless wave magnetic energy density in n-th resonance region (see Eq. 12). To solve numerically we have chosen the number density of alpha particles to be 5% of the proton density, that corresponds to mass-loading parameter being equal to 0.2. This He/H abundance ratio is typical for the fast solar wind streams (Gloeckler et al., 1994; Torsti et al., 2001; Desai and Burgess, 2008).

It should be noted that the heavy ions are not treated here as test particles, hence, the right hand side of the Eq. (5) for wave action was modified as well to include the input of the ions in wave generation and damping.

It should be mentioned that we use simple dispersionless form of pitch angle diffusion lines (3) for the sake of clarity. As the entire gyrophase averaged PDFs for both components are updated by Eqs. (15)–(17) it is rather straightforward to calculate self-consistently local thermal dispersive corrections to wave phase speed $v_{\rm ph}^{\leftrightarrow}$. The pitch-angle scattered distribution function will be more complicated than simple spherical shell in this case, but one should not expect substantial changes in the acceleration process. The effect of dispersion will be strongest for waves with frequencies at or close to a local proton cyclotron frequency $\omega_{\rm c}$, but resonances responsible for scattering of accelerated particles, especially heavy ions and high energy protons, are located at much lower frequency part of the wave spectrum.

In obtaining Eqs. (15)–(17) we introduced dimensionless magnetic variance

$$\tilde{B}^{2^{\rm dls}} = \tilde{B}^2/B_0^2, \tag{18}$$

where $\tilde{B}^2 = \sum |B_k|^2$ and used a normalized particle distribution function

$$f_{\rm p}^{\rm dls}\left(t,z,v_\parallel,v_\perp\right) = \frac{v_{\rm Tp}^3}{n_{\rm up}} f_{\rm p}\left(t,z,v_\parallel,v_\perp\right) \tag{19}$$

We also introduced dimensionless velocity, length, and time

$$\boldsymbol{v}^{\rm dls} = \frac{\boldsymbol{v}}{v_{\rm Tp}}, \quad t^{\rm dls} = \frac{t}{t_0}, \quad z^{\rm dls} = \frac{z}{z_0} \tag{20}$$

Here z_0 and t_0 are macroscopic length and time, connected by the relation $z_0 = v_{\rm Tup} t_0$. We omitted superscript "dls" in Eqs. (15)–(17).

To cover the possible interval of accelerated energies up to several MeV per nucleon, we choose the value of maximal velocity of protons as $v_{||\max} = 10^3$. The size of the parallel velocity interval was chosen as $\Delta v_{||} = 0.25$.

We used the following grid dimensions in numerical solution of Eqs. (4)–(5):

$$N_z = 100, \quad N_{v_{||}} = 4000, \quad N_{v_\perp} = 100 \tag{21}$$

and the number of harmonics in this study was $N_k = 10^6$. To assure that the size of each spatial interval in the region is much larger than the characteristic scale of quasi-linear relaxation the size L of domain in z direction was chosen to be $2\times 10^8 r_B$, where r_B is the proton gyroradius.

We used the magnetic field parameters similar to the case of ISEE-3 encounter with a quasi-parallel shock of 11–12 November 1978. The particle and wave observations for this event have been described in detail in previous publications (see Kennel et al., 1984a, b; Tsurutani et al., 1983). The magnetic field measured during the observations was $B_0 = 6.85$ nT.

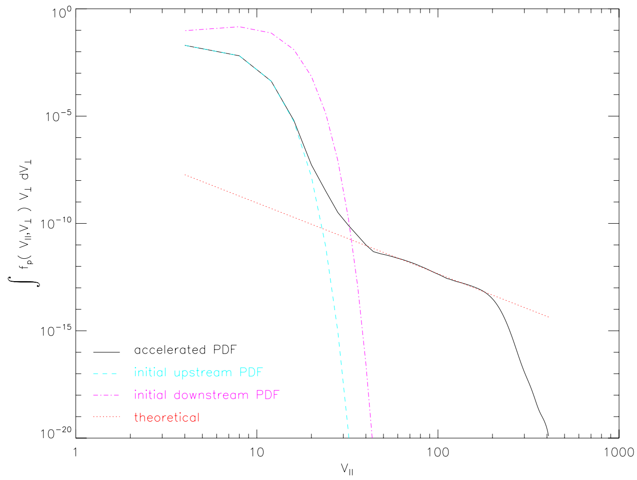

Fig. 1. A snapshot of the proton distribution function over parallel velocity $\int f_{\rm p}\left(t,z,v_\parallel,v_\perp\right) v_\perp dv_\perp$ typical for spatial/temporal area in the upstream region ($v_{\rm Tup} \approx 3$) close to the shock front. The dotted line is the line $v_\parallel^{-\beta}$ that represents the theoretical solution obtained by DSA approach ($\beta \approx 3.3$). The dashed and dash-dotted lines are initial Maxwellian upstream and downstream PDF, respectively.

For plasma parameters we used more strong shock than observed by ISEE-3. In our case the shock compression value was equal to $r = u_{\rm up}/u_{\rm d} = 10.4$. The solar wind plasma speed and density were $u_{\rm up} = 2.4\times 10^7\,{\rm cm\,s^{-1}}$ and $n_{\rm up} = 4\,{\rm cm^{-3}}$. Although the shock observed by ISEE-3 propogated obliquely to the magnetic field ($\theta \approx 41°$) we assumed paralalle shock propagation in this study to simplify the general picture (the original ISEE-3 observed shock strength and direction was used to analyze proton acceleration in a companion paper; Galinsky and Shevchenko, 2010).

The spectrum of the accelerated protons as a function of the parallel velocity is shown in Fig. 1 using a log-log scale. One can see a power-like spectrum of protons with the exponent close to $\beta = 3.3$ that is related to the compression value $\beta = 3r/(r\text{-}1)$. This result is in agreement with that of DSA-based theory (see e.g. Gordon et al., 1999), shown in the figure by dotted line $v_{||}^{-\beta}$. However, numerical solution of system of Eqs. (15)–(16) has shown that there is a break on the spectrum at large values of parallel velocity. The spectra of initial Maxwellian upstream and downstream PDFs are shown in the Figure as well with dashed and dash-dotted lines respectively to help understand the position of the break.

4 Discussion

Although the right-hand-side terms in Eqs. (4)–(5) (or 15–16) can be obtained without using quasi-linear framework and hence can be made applicable even to large amplitude monochromatic wave regime, the quasi-linear approach used in this paper for simplification imposes some important restrictions. First of all, in order to be applicable the quasi-linear approach requires that the parallel velocity grid points inside each resonant interval should be close enough to allow the overlapping of the trapping regions of neighboring harmonics. The expression for the distance $\delta v_{||}$ between the trapping regions of two neighboring harmonics has the form:

$$\delta v_{||} < \left\{\Omega_{\mathrm{cp}} k v_{\perp}\right\}^{1/2}/k \tag{22}$$

where $\Omega_{\mathrm{cp}} = e\tilde{B}/mc$, $\tilde{B} = \left(|B_k|^2 \delta k\right)^{1/2}$ is a root mean square magnetic field of the wave harmonic, $v_{\perp} \approx v_T$ is characteristic perpendicular velocity, δk is the wave number distance between neighboring harmonics, and $|B_k|^2$ is a spectral energy density of the electromagnetic fluctuations.

By using cyclotron resonance condition it follows that

$$\delta v_{||} < \left(\frac{e^2|B_k|^2}{m^2c^2}\frac{v_T^2}{\omega_{\mathrm{c}}}\right)^{1/3} \tag{23}$$

And hence, the width of each resonant interval should satisfy the condition:

$$\Delta v_{||} \gg \delta v_{||} \tag{24}$$

The conditions (23)–(24) have been checked to be satisfied in our study.

An interesting feature revealed by the model can clearly be seen in Figure 1. It is the presence of a spectral break. The break is located at the high energy part of the particle power spectrum, where acceleration evidently stops and the PDF of accelerated particles (solid line) deviates from the DSA results (dotted line). The non-stationary nature of the model can be used as one possible explanation for the origin of the break. Indeed, we found that the position of the break depends both on distance from the shock front and on time elapsed since the acceleration process has started. As acceleration to higher energies requires more time it is not clear yet if asymptotic stationary solution without the break can be reached in a finite time for any shock configurations or, on the contrary, the position of the break will stabilize at some point in space and time.

In Fig. 2 the updated distribution function of alpha particles (see procedure 1 of *the algorithm*) in the upstream region close to the wave front is shown at the beginning of the first time step. This is a plasma-beam distribution with beam component formed by the tail particles of the downstream distribution that crossed the shock front. This distribution is unstable with respect to excitation of MHD waves that are needed for the resonant particles to change their velocities. As a result, some particles can cross the shock front repeatedly and eventually be accelerated. Hence, the solution demonstrates that neither seed population nor initial waves are needed for the acceleration process to start. To emphasize the importance of this finding we would like to stress once more that all previous DSA-based macroscopic models needed a seed population for description of acceleration process at shocks. Of course, there was general understanding that "seed" population is injected from the thermal background (see e.g. Drury, 1983; Jones and Ellison, 1991). Malkov and Völk (1995, 1998) have shown that the tail particles from downstream thermal distribution penetrating into upstream region and exciting waves needed for particle acceleration can play the role of this seed population. However, DSA-based macroscopic models cannot use this mechanism because they are limited by derivation to energetic particles only. We would like to note that Malkov-Völk mechanism is automatically incorporated in our macroscopic model.

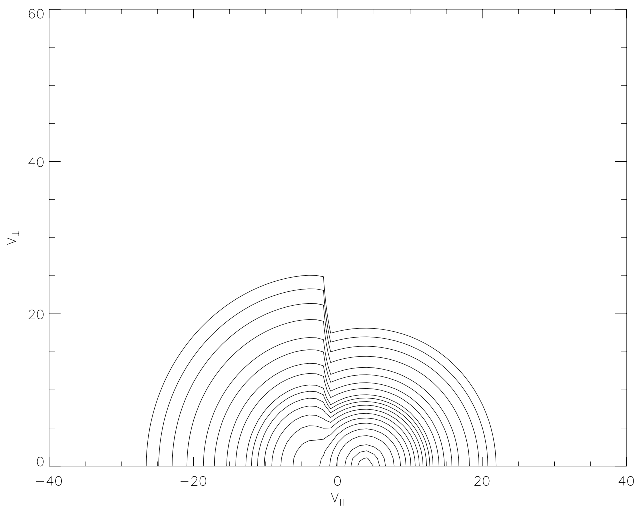

Fig. 2. The updated distribution function of alpha-particles at the beginning of the first time step in the upstream region close to the shock front.

The model showed a substantial difference in acceleration of heavy ions from test particles regime. It can be seen from Fig. 3, that even for relatively small concentration of alpha particles (5%) the number of accelerated alpha particles is much larger when the hydromagnetic turbulence is generated by both protons and alphas themselves, than in the case where heavy ions are treated using a test particle approximation.

We would like to mention several of the most important restrictions of our model in the form presented here (besides the quasi-linear derivation of right hand sides of Eqs. 4 and 5). First of all, the model is one dimensional: movement of plasma particles as well as excitation and propagation of waves are confined to one direction. This limitation is not very restrictive for the problem of heavy ions/protons acceleration by parallel shocks considered here and in Paper 1, but may be more important for problem of acceleration by oblique shocks (Galinsky and Shevchenko, 2010).

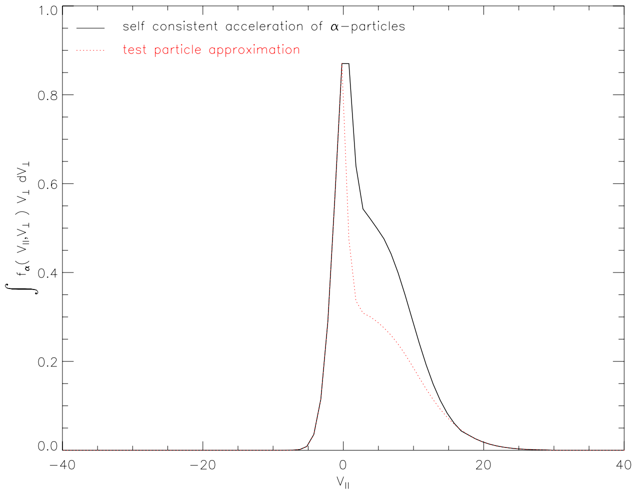

Fig. 3. The plot of alpha-particle distribution function over parallel velocity in the upstream region (units are arbitrary).

Second, similarly to DSA-based models, only resonant wave-particle interactions were included in the model. Including a non-resonant interaction represents a challenging problem and we defer discussion of possible effects of this interaction to a subsequent paper.

Another unsolved problem in the study of shock acceleration is the role of nonlinear wave-wave interactions in the upstream region. The parametric interaction of Alfvén and acoustic waves can be described in the framework of the so-called derivative nonlinear Schrodinger (DNLS)-type equation (Shevchenko et al., 2002). Using this description it was shown (Shevchenko et al., 2003) that nonlinear interaction can lead to the development of short large-amplitude magnetic structures (SLAMS) that were observed in the solar wind by Schwartz et al. (1992). Such nonlinear wave structures can reflect protons (see e.g. Claßen and Mann, 1998) and thus provide energy for downstream heating and wave excitation. Our approach permits the inclusion of wave-wave interaction in the shock acceleration model. We will consider these questions in detail elsewhere.

5 Conclusions

The theoretical study of alpha particle acceleration at a shock due to interaction with Alfvén waves self-consistently excited in both upstream and downstream regions was conducted using a new theoretical scale-separation model. The main difference of the model from DSA models is that it does not treat the accelerated particles as a separate substance but, similar to shock simulations, considers them as an integral part of the plasma distribution function. The model automatically includes an injection scenario in the macroscopic picture of the particle acceleration at shocks and confirms that neither an additional population of suprathermal seed particles nor external wave turbulence are required for the acceleration process to operate. The numerical analysis based on the model shows quite good agreement with DSA models in predicting power spectra of accelerated particles. However, it was shown that there is a break in the power spectra at large energies of particles. It was shown that in the case where the hydromagnetic turbulence is self-consistently generated by both protons and alphas, the number of accelerated alpha particles is much larger than in the case when they are treated using a test particle approximation.

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