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Nonlinear wave energy modelling in the surf zone

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Abstract. Breaking wave energy in the surf zone is modelled through the incorporation of the time dependent energy balance equation in a non linear dispersive wave propagation model. The energy equation is solved simultaneously with the momentum and continuity equation. Turbulence effects and the non uniform horizontal velocity distribution due to breaking is introduced in both the energy and momentum equations. The dissipation term is a function of the velocity defect derived from a turbulent analysis. The resulting system predicts both wave characteristics (surface elevation and velocity) and the energy distribution inside surf zone. The model is validated against experimental data and analytical expressions.

1 Introduction

Wave energy dissipation due to breaking is one of the most important processes in the nearshore zone. Since no theory yet exists for the description of the breaking and the dissipation after breaking of the non linear waves an extension of the existing wave propagation models is generally adopted. One type of wave breaking propagation models is based on Boussinesq equation (Karambas and Koutitas, 1992, Sato et al., 1992, Schaffer et al. 1993). These models, using different methods, can describe the dissipation after breaking and predict the surface elevation and horizontal velocity. However information are required for the wave energy (and its dissipation) distribution in the surf zone to be used as input in a wave induced current model (Pechon, 1991, Deigaard et al., 1991).

Other models use the wave energy equation for the prediction of wave height decay in the surf zone. The equation, in a period averaged form, is written as:

$$\frac{d\left(c_g E\right)}{dx} = D \tag{1}$$

in with c_g is the group velocity, E the wave energy and D its dissipation.

Linear theory is generally used for the estimation of E while the estimation of the dissipation D is based on the similarity between a hydraulic jump and breaking waves (Stive, 1984) or on the assumption that the dissipation rate is proportional to the difference between the local energy and the stable energy flux (Dally et al., 1984). However, non-linear processes in shallow water are very significant and can not be ignored. In addition, turbulence and the vertical distribution of the horizontal velocity profile also play an important role in the energy and momentum balance and must be included. Recently, Peronnard and Hamm (1994) presented a new approach based on the non-linear wave energy balance and the inclusion of the surface roller effect. In a similar way, instead of the use of non linear wave theory, a non-linear time dependent breaking wave propagation model can also be employed. In addition these type of models can also predict time series of the surface elevation and the velocity, a significant advantage.

The aim of this work is to incorporate into the energy balance equation the effects of non-linearity, turbulence and the non-uniform velocity distribution. The time dependent wave energy equation is solved in connection with a non-linear dispersive wave model. Since it includes additional turbulent terms more than the momentum equation, such as the dissipation term (Rouse et al., 1958), its incorporation into a continuity-momentum system is also expected to give a proper description of the wave breaking and more information for the prediction of the wave characteristics inside surf zone. The horizontal velocity distribution which is introduced in the system, is based on the similarity in the flow between hydraulic jumps and breaking waves (Madsen and Svendsen, 1983). The estimation of the dissipation also based on the above similarity and is a function of the defect velocity as in turbulent wake theory (Tennekes and Lumley, 1972). The approach is based on the Svendsen and Madsen (1984) model (hereinafter designated S-M) for the extension of the Shallow Water Equations (SWE) to include the effects of turbulence in a bore propagation problem. Instead of the SWE, a non linear dispersive wave propagation model based on the Boussinesq equations is used.

2 Basic Equations

The momentum equation in a Boussinesq model can be written in the form(Peregrine, 1972, Karambas and Koutitas, 1992, Schaffer et al., 1993):

$$\frac{\partial U}{\partial t}+\frac{1}{h}\frac{\partial\int_{-d}^{\zeta}u^2dz}{\partial x}-\frac{1}{h}\frac{U\partial(Uh)}{\partial x}+g\frac{\partial\zeta}{\partial x}=$$

$$\frac{d}{2}\frac{\partial^3(Ud)}{\partial x^2\partial t}-\frac{d^2}{6}\frac{\partial^3 U}{\partial x^2\partial t}+\frac{1}{h}\frac{\partial}{\partial x}\left(v_t h\frac{\partial U}{\partial x}\right) \tag{2}$$

where u=u(z) is the horizontal velocity, ζ is the surface elevation, v_t is the eddy viscosity coefficient for the simulation of Reynolds stresses, d is the still water depth, $h=d+\zeta$, and the quantity U(x,t) is defined as:

$$U=\frac{1}{h}\int_{-d}^{\zeta}u\,dz$$

The equation of the conservation of the energy density E of the mean flow per unit horizontal area is written (Madsen and Svendsen, 1979, S-M model):

$$\frac{\partial E}{\partial t}+\frac{\partial E_f}{\partial x}=D \tag{3}$$

with

$$E=\int_{-d}^{\zeta}\frac{1}{2}u^2dz+\frac{1}{2}g\zeta^2 \tag{4}$$

$$E_f=\int_{-d}^{\zeta}\frac{1}{2}u^3dz+g\zeta\,Uh \tag{5}$$

and D is the dissipation of the mean energy (equal to minus the production of turbulent energy) defined as:

$$D(x,t)=\int_{-d}^{\zeta}\overline{u'w'}\frac{\partial u}{\partial z}dz+\int_{-d}^{\zeta}\left(\overline{u'^2}-\overline{w'^2}\right)\frac{\partial u}{\partial x}dz \tag{6}$$

where u' and w' are the horizontal and vertical turbulent intensities.

In the S-M model the pressure is supposed to be hydrostatic. In the present model dispersive wave propagation (whether breaking or not) is considered as a non-hydrostatic term which is introduced similar to the dispersion term of the Boussinesq equations (see also Whitham, 1965):

$$Uh\frac{d^2}{3}\frac{\partial^3 U}{\partial x^2\partial t} \tag{7}$$

In this way the energy equation (3) has a form similar to the Boussinesq momentum equation in order to avoid consistency problems in the numerical solution. Finally, after the elimination of the integral in equation (2) and using equation (4) the system of the three equations (continuity, momentum and energy) is written in the form:

$$\frac{\partial\zeta}{\partial t}+\frac{\partial(Uh)}{\partial t}=0$$

$$\frac{\partial U}{\partial t}+\frac{1}{h}\frac{\partial(2E-g\zeta^2)}{\partial x}-\frac{1}{h}\frac{U\partial(Uh)}{\partial x}+g\frac{\partial\zeta}{\partial x}=$$

$$\frac{d}{2}\frac{\partial^3(Ud)}{\partial x^2\partial t}-\frac{d^2}{6}\frac{\partial^3 U}{\partial x^2\partial t}+\frac{1}{h}\frac{\partial}{\partial x}\left(v_t h\frac{\partial U}{\partial x}\right)$$

$$\frac{\partial E}{\partial t}+\frac{\partial E_f}{\partial x}=D+Uh\frac{d}{2}\frac{\partial^3(Ud)}{\partial x^2\partial t}-Uh\frac{d^2}{6}\frac{\partial^3 U}{\partial x^2\partial t} \tag{8}$$

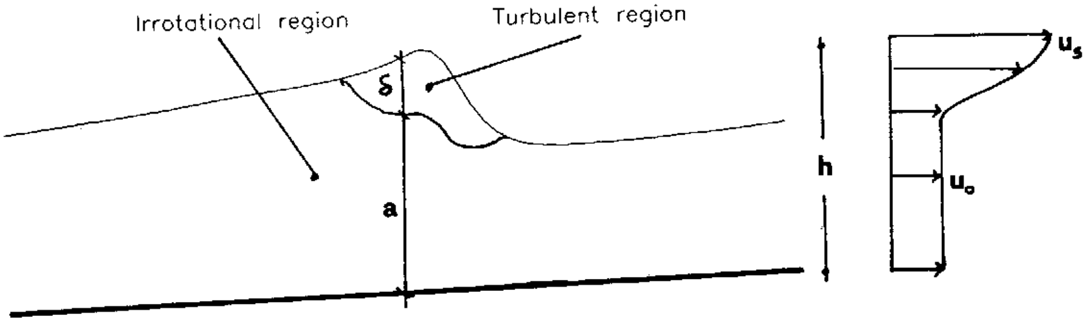

Fig. 1. Definition sketch: turbulent and irrotational region of a breaker inside the inner region and corresponding horizontal velocity distribution. Equation (9) is applied in the turbulent region while equation (10) in the irrotational region.

Assuming a similarity in the flow between an hydraulic jump and a breaking wave in the inner region, a non-uniform velocity distribution is adopted (Madsen and Svendsen, 1983) to describe the flow in the turbulent region δ=h-a, (Fig. 1):

$$u(z)=u_o+u_d\, f(\sigma) \qquad \text{for } \zeta-\delta<z<\zeta \qquad (9)$$

with

$$u_d = u_s - u_o$$

$$f(\sigma)=-A\sigma^3+(1+A)\sigma^2, \quad \sigma=(d+z)/h, \quad A=1.4$$

where u_s is the surface velocity and u_o the velocity at the lower part of the wave at z=-(d-a)

In the S-M model the flow in the non-turbulent region is simulated by another momentum equation and the velocity u_o is taken to be constant over the depth. In the present model, since we not consider a single bore propagation but breaking waves, another velocity simulation in the lower region is adopted. Nadaoka et al. (1989), in their experimental study of breaking waves, separate the velocity u into a rotational part u_r and an irrotational part u_p and find that in the non-turbulent region (fig. 1 and fig. 20 of their paper) u_r is not significant and u is close to irrotational value u_p and approximately constant. Here we assume that in this nearly uniform velocity region the irrotational expression for near bottom velocity given by Peregrine (1972) can be applied:

$$u_o = U + \frac{d^2}{6}\frac{\partial^2 U}{\partial x^2} \qquad \text{for } -d<z<\zeta-\delta \qquad (10)$$

Nadaoka et al. (1989) also used non linear irrotational theory to estimate the velocity field from the surface elevation measurements of a breaking wave.

Using the definition of the mean velocity U and equation (9) the turbulent region depth δ can be estimated from:

$$\delta = \frac{U-u_o}{u_d}\frac{h}{0.45} \qquad (11)$$

applied in a length detected geometrically as in Schaffer et al. (1993).

Another empirical estimation of the turbulent length is to consider the turbulent region where $\partial^2\zeta/\partial x^2<0$. Both values of δ are introduced in the model without giving significant differences in the model results. In figure 2 the turbulent regions of a breaker are shown in a position 1 meter shoreward from the point from which the inner region begins. The region length is detected geometrically as in Schaffer et al. (1993) but its depth δ is given from (11). Schaffer et al. (1993) used an empirical coefficient for the estimation of δ.

The surface velocity u_s is taken equal to the celerity c (Schaffer et al., 1993):

$$u_s = c \qquad (12)$$

while the energy flux E_f is estimated using the velocity distribution in equation (9) and equation (5):

$$E_f=0.5\, h\, u_o^3 +0.5\, \delta\, (0.24\, u_d^3+1.35\, u_o^2\, u_d+0.936\, u_o^2\, u_d^2) + g\, \delta\, h\, U \qquad (13)$$

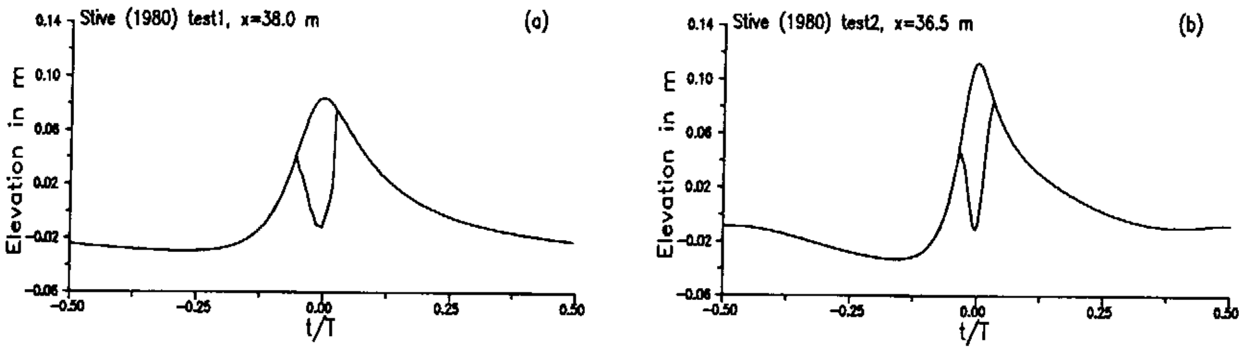

Fig. 2. Turbulent region δ of breaking wave inside the inner region: (a) spilling breaker (test 1) ; and (b) plunging breaker (test 2).

The dissipation of the wave energy in equation (8) is given by Madsen and Svendsen (1983) and Svendsen and Madsen (1984), after neglecting the second term of the right side part of the equation (6) and the work done by Reynolds stresses:

$$D=-\Omega(u_s-u_o)^3 \qquad (14)$$

Equation (14) is derived from an analytical solution of the turbulent kinetic energy equation. The assumption of a local equilibrium in k (valid in the roller region where u_s=c) has also been used to derive equation (14). A similar expression for D is given by Tennekes and Lumley (1972) for turbulent wakes. The value of the turbulence constant Ω is taken equal to 0.015 (Ω=0.015), close to that proposed by Madsen and Svendsen (1983) and Svendsen and Madsen (1984).

The eddy viscosity coefficient ν_t is estimated from a simplified time dependent turbulent model based on the turbulent kinetic energy equation (Karambas and Koutitas, 1992). The turbulent constant C_d is taken equal to 0.08 (C_d=0.08), the characteristic length of Energy Containing Eddies l_c=0.3d and the constant M=1. In this way a much smaller eddy viscosity is predicted instead of the unrealistic value proposed by Karambas and Koutitas (1992). The production term is taken equal to the dissipation of the wave energy term given in equation (14).

3 Numerical Solution and Applications

The following algorithmic structure is used in the numerical computations:

1. Calculation of U^{n+1} , ζ^{n+1} (at time level (n+1)Δt) from the solution of the Boussinesq equations (continuity and momentum).
2. Calculation of E^{n+1} *explicitly* from the energy equation using exactly the same Finite Differences approximations (both new time step (n+1)Δt and previous time step nΔt values of the variables are employed).
3. Next time step: $U^n = U^{n+1}$, $\zeta^n = \zeta^{n+1}$ and $E^n = E^{n+1}$ and replacement in the non linear term of the momentum Boussinesq equation the values E^n and ζ^n in the non-linear term of the momentum Boussinesq equation.

In this manner energy equation is numerically solved simultaneously with the Boussinesq equations and its effects are introduced explicitly in the momentum equation. The main advantage of the above procedure is that it can be easily introduced in the existing models without the need for changing their numerical scheme.

In the non-turbulent region, D=0, $E=0.5hU^2+0.5g\zeta^2$ and the system reduces to the classical Boussinesq equations.

Experimental data from Stive (1983) are used here for comparison with numerical simulations. The data were conducted in a wave flume, 55 m long, 1 m wide and 1 m high. Regular waves without the free second harmonic were generated in a water depth of 0.85 m. The depth is reduced in the horizontal section to 0.70 m. The waves broke on a plane beach with a 1:40 slope. The wave conditions were:

<u>test 1</u>: H =0.1586 m and T=1.79 sec
<u>test 2</u>: H =0.1410 m and T=3.00 sec

Surface elevations were measured by means of conductivity-type wave gauges while the horizontal and vertical component of velocity were measured by means of a laser doppler velocimeter (LDV). LDV measurements are not possible in the roller region of the breakers. Therefore extrapolations were made of the flow field in the crest region. The extrapolation of the periodic horizontal velocity field is based on the balance equation of mass while for the periodic vertical velocity the kinematics boundary condition at the free surface is used. The method was satisfactorily checked in the trough region. The above procedure introduced an inaccuracies, in the mean kinetic energy data, of a few per cent.

Surface elevation profiles inside surf zone are compared with measurements in Fig. 3 to 6. The model predictions agree well with experimental results. Other models (Karambas and Koutitas, 1992, Schaffer et. al., 1993) can also give similar results as far as the surface elevation is concerned. However the present model gives also information about the wave energy distribution (and its dissipation) inside surf zone. The breaking wave energy and its dissipation are used as input in the wave induced current models for both the estimation of radiation stress and the production term of the turbulent kinetic energy equation (Pechon, 1991, Deigaard et al., 1991). Usually the models are based on the linear wave energy balance equation in a period averaged form. Thus the results are also averaged over the period and empirical expressions are needed for the prediction of their temporal variation (as in Deigaard et al., 1991).

In Fig. 7 and 8 the model results for wave energy flux E_f (a period averaged value) are tested against Stive measurements. The comparison also shows good agreement.

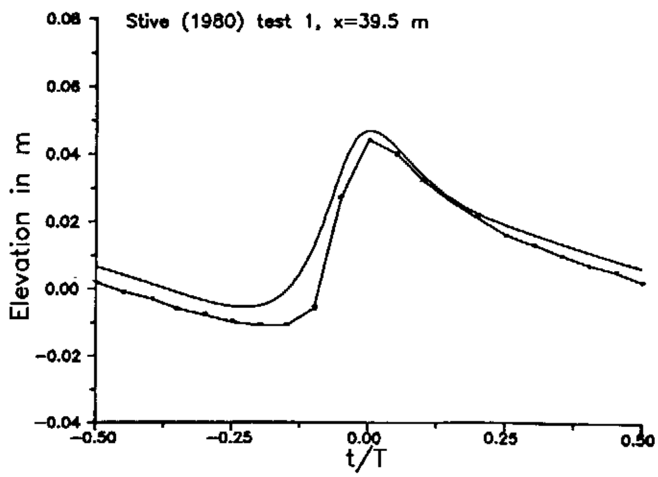

Fig. 3. Comparison of the model simulations for surface elevation ζ with the measurements: Stive, 1983, Test 1, x=39.5 m. (•——•) data, (——) model results.

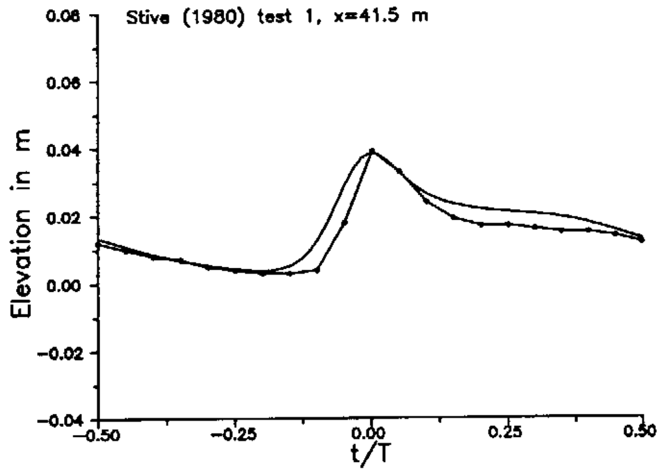

Fig. 4. Comparison of the model simulations for surface elevation ζ with the measurements: Stive, 1983, Test 1, x=41.5 m. For caption see Fig. 3.

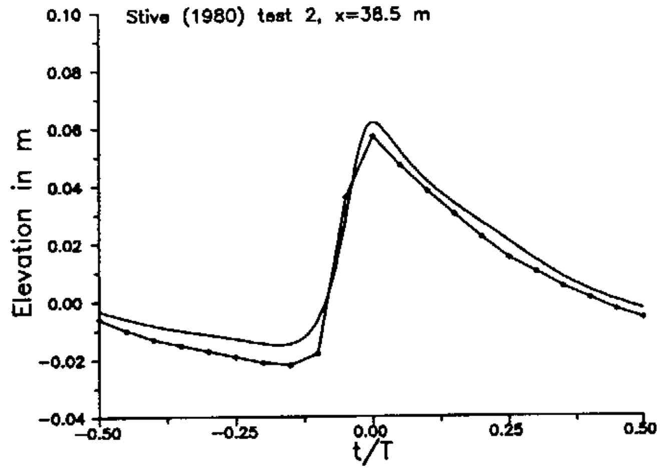

Fig. 5. Comparison of the model simulations for surface elevation ζ with the measurements: Stive, 1983, Test 2, x=38.5 m. For caption see Fig. 3.

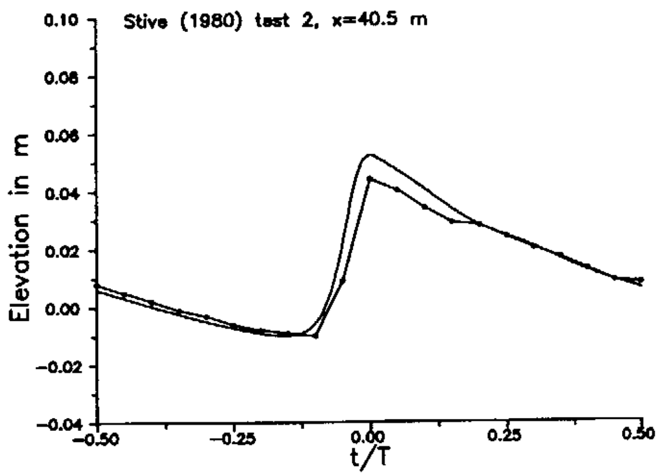

Fig. 6. Comparison of the model simulations for surface elevation ζ with the measurements: Stive, 1983, Test 2, x=40.5 m.

The two most frequently used approaches for the estimation of the wave energy dissipation are compared with both the present model and experimental data. Both models are based on linear theory but the expression of the dissipation is different. In the first model the dissipation proposed by Stive (1984), and also used by many other researchers, is adopted:

$$D = -\frac{1}{4}\rho g c \frac{H^3}{dL} \qquad (15)$$

with L the wave length, H the height and c the celerity. The above relation is derived from the hydraulic jump dissipation (Lamb, 1945).

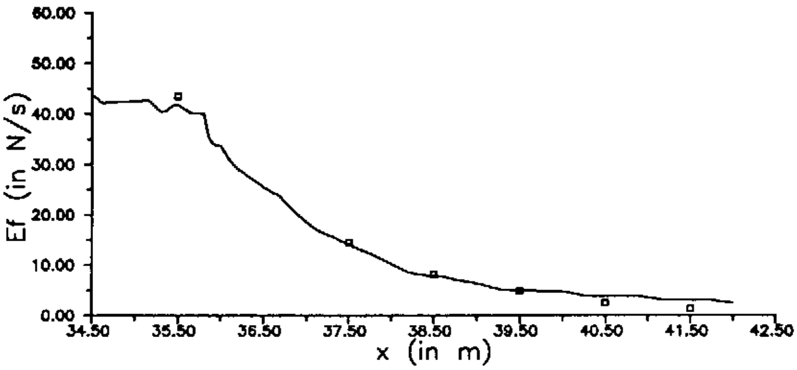

Fig. 7. Comparison between calculated and measured values of the wave energy flux inside the surf zone: Stive, 1983, Test 1. (□ □ □) data, (———) model results.

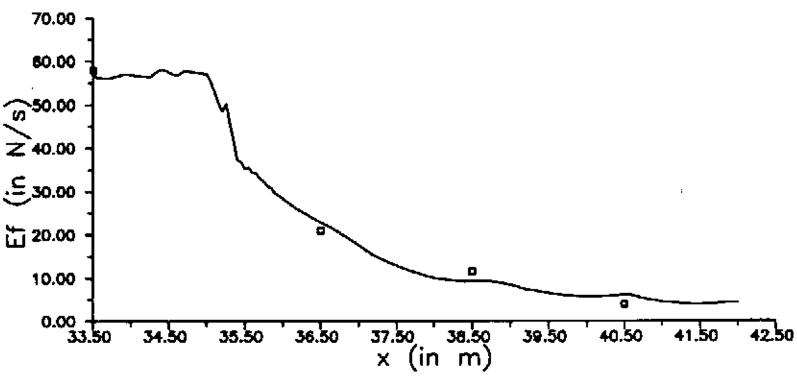

Fig. 8. Comparison between calculated and measured values of the wave energy flux inside the surf zone: Stive, 1983, Test 2. For caption see Fig. 7.

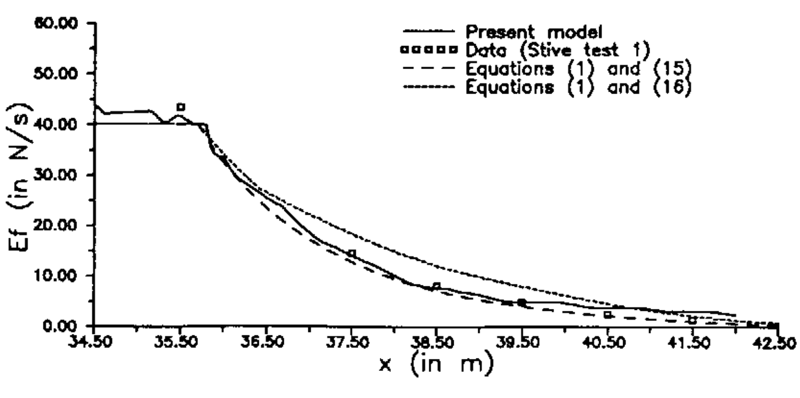

Fig. 9. Comparison between results from three different models and measurements of the wave energy flux inside the surf zone: Stive, 1983, Test 1.

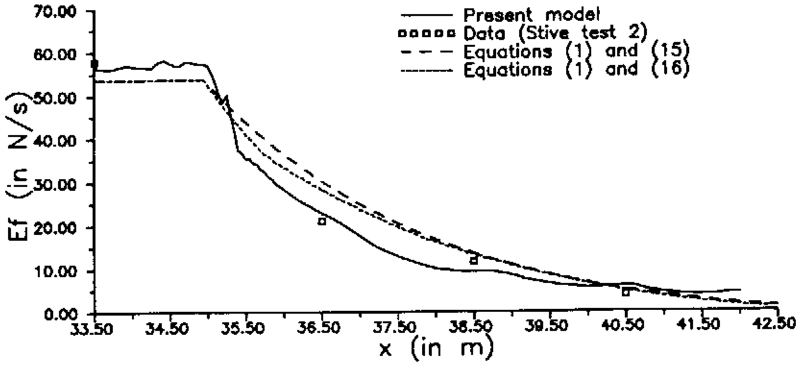

Fig. 10. Comparison between results from three different models and measurements of the wave energy flux inside the surf zone: Stive, 1983, Test 2.

In the second model the relation proposed by Dally et al. (1984) is used:

$$D = -\frac{K}{d}\left[Ec_g - \left(Ec_g\right)_s\right] \tag{16}$$

with K=0.15 and $(Ec_g)_s$ the energy flux associated with a stable wave H_s=0.4d.
Equation (1) is integrated numerically (Oelerich and Dette, 1988), taking into account the mean water level effects (set-up and set-down).

The results of the comparison between the above approaches and the present non-linear model are shown in Fig. 9 and 10. The present method gives a better comparison. Stive (1984) and Svendsen (1984) proposed the introduction of two different coefficients in equation (15) in order to improve the results. However both coefficients are estimated empirically, depending on the wave characteristics, and are not used here. The above comparison is also an indirect way to check the validity of equation (14) for D, against equations (15) and (16), which are generally applied. Another reason for this deviation could be the use of a non uniform horizontal velocity distribution (surface roller effects) and also the non-linearity (as shown in the shoaling-region, linear-theory underestimates the energy flux).

Although only periodic wave results are presented the model can also be used for irregular waves without changes (as in Schaffer et al., 1993) since the parameters which involved here are the same for different wave conditions i.e. for the spilling breaker of test 1 and the plunging breaker of test 2.

4 Conclusion

Wave energy dissipation inside the surf zone can be modelled though the incorporation of the energy equation into a non-linear breaking-wave propagation system based on Boussinesq equations.

The simultaneous solution of the energy, momentum and continuity equations results not only in the extension of a non-linear dispersive wave model in the surf zone, but also to the non linear wave energy distribution. In this way the model can be used both for breaking-wave characteristics (i.e. surface elevation, velocity, wave height, set-up) and the prediction of the dissipation D. The first results are used in many engineering applications such as the design of coastal structures. On the other hand the relation for the dissipation D (equation 14) can be used instead of the period averaged relations (equations 15 and 16). This information is very useful in modelling the breaking wave induced current since D is the production term of the turbulent kinetic energy equation.

The comparison with other models shows the importance of including the non-linear terms, using a Boussinesq model, as well as the surface roller effects through the introduction of a non uniform velocity field over the depth averaged horizontal velocity.

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